ECE 602 Homework 2

April 5, 2011

1 Question 1

Part A:

The block diagram for the series system can be found in the appendix.
The state space realization is as follows:
The state space equation is

$$x_1(t) = A_1x_1 + B_1(C_2x_2 + D_2u(t))$$

$$y_1 = C_1x_1 + D_1y_2 = C_1x_1 + D_1(C_2x_2 + D_2u(t))$$

Also, the state space equation for the second set include:

$$x_2(t) = A_2x_2 + B_2u(t)$$

$$y_2 = C_2x_2 + D_2u(t)$$

These can be transformed into matrix form:
Matrix A is:

$$\begin{pmatrix} A_1 & B_1C_2 \\ 0 & A_2 \end{pmatrix}$$

Matrix B is:

$$\begin{pmatrix} B_1D_2 \\ B_2 \end{pmatrix}$$

Matrix C is:

$$\begin{pmatrix} C_1 & D_1C_2 \\ 0 & C_2 \end{pmatrix}$$

Matrix D is:

$$\begin{pmatrix} D_1D_2 \\ D_2 \end{pmatrix}$$

Part B:

The block diagram for the parellel system can be found in the appendix.
Matrix A is:

$$\begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}$$

Matrix B is:

$$\begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$$

Matrix C is:

$$\begin{pmatrix} C_1 & C_2 \end{pmatrix}$$

Matrix D is:

$$\begin{pmatrix} (D_1 + D_2) \end{pmatrix}$$

Part C
The block diagram for the feedback system can be found in the appendix.
The state space realization was found to be:

$$x_1 = [A_1 - \frac{B_1 D_2 C_1}{1 + D_1 D_2}]x_1 + [B_1 - \frac{B_1 D_2 D_1}{1 + D_1 D_2}]u(t) + [\frac{B_1 D_2 D_1 C_2}{1 + D_1 D_2} - B_1 C_2]x_2$$

$$x_2 = [A_2 - \frac{B_1 D_1 C_2}{1 + D_1 D_2}]x_2 + [fracB_2 C_1(1) + D_1 D_2]x_1 + [\frac{B_2 D_1}{1 + D_1 D_2}]u(t)$$

$$y(t) = \frac{1}{1 + D_1 D_2}(C_1 x_1 + D_1 u(t) - D_1 C_2 x_2)$$

2 Question 2

(Please note that the thetas that are raised to the 1st, 2nd, and 3rd powers are representing the number of dots for that particular figure. Could not find proper command to display)

$$theta^1 = 1dot$$
$$theta^2 = 2dots$$
$$theta^3 = 3dots$$

Given

$$\dot{x} = [\theta\theta^1\theta^2]x(t) + [\theta^3 + \theta)u(t)$$
$$H(s) = \frac{Y(s)}{X(s)}$$

In order to compute the transfer function Y(s) and X(s) must be found.

$$= \frac{\theta}{\theta\theta^1\theta^2 + \theta^3 + \theta}$$
$$= \frac{1}{\theta}$$

3 Question 3

Part A: It was found in the problem that

$$x(k) = \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{pmatrix}$$

$$x(k+1) = \begin{pmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{pmatrix} = \begin{pmatrix} x_1(k) + x_2(k) + x_3(k)/3 \\ x_1(k) + x_2(k) + x_3(k)/3 \\ x_1(k) + x_2(k) + x_3(k) + x_4(k)/4 \\ x_3(k)x_4(k)/2 \end{pmatrix}$$

$$\begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{pmatrix}$$

The 4x4 A matrix is:

$$A = \begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}$$

Part B

The eigenvalues for matrix A are: 0.995, 0.562, 0, -0.147 The eigenvectors are:

$$v_1 = \begin{pmatrix} 0.495) \\ 0.495 \\ 0.502 \\ 0.508 \end{pmatrix}$$

$$v_2 = \begin{pmatrix} 0.360 \\ 0.360 \\ -0.107 \\ -0.854 \end{pmatrix}$$

$$v_3 = \begin{pmatrix} -0.707 \\ 0.707 \\ 0 \\ 0 \end{pmatrix}$$

$$v_4 = \begin{pmatrix} -0.294 \\ -0.294 \\ 0.720 \\ -0.556 \end{pmatrix}$$

Part C It is already stated that: x(k+1) = Ax(k) which leads to x(1) = Ax(0) x(2) = Ax(1) = AAx(0)

$x(k) = A^k x(0)$

Also, with k=1

$$w_1^T x(1) = w_1^T x(0) - -- > w_1^T A^1 x(0) = w_1^1 x(0)$$

Now

$$w_1^T A^1 = w_1^T$$

must be solved:

$$A = \begin{matrix} 1/4 & 1/4 & 1/3 & 1/6 \end{matrix}$$

is multiplied times

$$\begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}$$

=

$$A = \begin{matrix} 1/4 & 1/4 & 1/3 & 1/6 \end{matrix}$$

It was found that this does hold

$$w_1^T A^1 = w_1^T$$

The final yield is:

$$W_1^T x(k) = w_1^T x(0) = w_1^T x(0)$$

Part D

$$e(k) = x(k) - [w_1^T x(0)]$$

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} x_1(k) - \frac{1}{4}x_1(k) - \frac{1}{4}x_2(k) - \frac{1}{3}x_3(k) - \frac{1}{6}x_4(k) \\ x_2(k) - \frac{1}{4}x_1(k) - \frac{1}{4}x_2(k) - \frac{1}{3}x_3(k) - \frac{1}{6}x_4(k) \\ x_3(k) - \frac{1}{4}x_1(k) - \frac{1}{4}x_2(k) - \frac{1}{3}x_3(k) - \frac{1}{6}x_4(k) \\ x_4(k) - \frac{1}{4}x_1(k) - \frac{1}{4}x_2(k) - \frac{1}{3}x_3(k) - \frac{1}{6}x_4(k) \end{pmatrix}$$

=

$$A = \begin{matrix} 3/4 & -1/4 & -1/3 & -1/6 \\ -1/4 & 3/4 & -1/3 & -1/6 \\ -1/4 & -1/4 & 2/3 & -1/6 \\ -1/4 & -1/4 & 1/3 & 5/6 \end{matrix}$$

multiplied times

$$\begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{pmatrix}$$

$$Byutilizingx(k) = A^k x(0) = A * A^k - 1x(0)$$

= e(k)

$$\begin{matrix} 3/4 & -1/4 & -1/3 & -1/6 \\ -1/4 & 3/4 & -1/3 & -1/6 \\ -1/4 & -1/4 & 2/3 & -1/6 \\ -1/4 & -1/4 & 1/3 & 5/6 \end{matrix}$$

multiplied times

$$\begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}$$

$$\begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}^{k} - 1$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{pmatrix}$$

Utilizing computational calculations it is found:
= e(k)

$$\begin{matrix} 0.0833 & 0.0833 & 0 & -0.1667 \\ 0.0833 & 0.0833 & 0 & -0.1667 \\ 0 & 0 & -0.0833 & 0.0833 \\ -0.2500 & -0.2500 & 0.1667 & 0.3333 \end{matrix}$$

multiplied times

$$\begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}^{k} - 1$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{pmatrix}$$

Since all the elements in the matrix above are between numerical -1 and 1, as k approaches infinite

$$B * A^k - - > 0 soe(k) does approach 0$$

Part E

The exponential rate at which e(k)—¿0 is found be 1.

Part F

Utilizing the symmetric matrix P such that

$$f(x) = x^T P x$$

f(x) =

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}^T$$

multiplied times

$$\begin{matrix} a11 & a12 & a13 & a14 \\ a21 & a22 & a23 & a24 \\ a31 & a32 & a33 & a34 \\ a41 & a42 & a43 & a44 \end{matrix}$$

multiplied times

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$$

$$By allowing P_1, P_2, P_3, and P_4 to be the columns of matrix P the following should be found$$

$$f(x) = [x^T P_1 x^T P_2 x^T P_3 x^T P_4][x1x2x3x4]^T$$

$$= x^T P_1 x_1 + x^T P_2 x_2 + x^T P_3 x_3 + x^T P_4 x_4$$
$$= a11x_1^2 + a21x_2x_1 + a31x_3x_1 + a41x_4x_1 + a12x_1^2 + a22x_2^2 + a32x_3x_2 + a42x_4x_2$$
$$+a13x_1x_3 + a23x_2x_3 + a33x_3^2 + a43x_4x_3 + a14x_1x_4 + a24x_2x_4 + a34x_3x_4 + a44x_4x_4^2$$

$$= a11x_1^2 + (a21 + a12)x_2x_1 + (a31 + a13)x_3x_1 + (a41 + a14)x_4x_1$$
$$+a22x_2^2 + (a32 + a23)x_3x_2 + (a42 + a24)x_4x_2 + a33x_3^2 + (a43 + a34)x_4x_3 + a44x_4^2$$

The above equation can be referred to as 2a

$$Since it is known that matrix P is symmetric, P = P^T then equation 2a can be simlified to:$$

$$f(x) = a11x_1^2 + 2(a21)x_2x_1 + 2(a31)x_3x_1 + 2(a41)x_4x_1$$
$$+a22x_2^2 + 2(a32)x_3x_2 + 2(a42)x_4x_2 + a33x_3^2 + 2(a43)x_4x_3 + a44x_4^2$$

Using the two similarity matrices found above, matrix can be determined to be:

$$\begin{matrix} 1 & -1 & -1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{matrix}$$

Part G

By utilizing theorem 3.7 from "An Introduction to Optimization", it was determined by f(x) ¿ or = 0 by looking at the eigenvalues of matrix P.

The eigenvalues of P were found to be

-1.1701, 0.6889, 2, 2.4812

Which makes P indefinite.

Part H

$$\begin{matrix} 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/3 & 1/3 & 0 \\ 1/3 & 1/4 & 1/4 & 1/4 \\ 0 & 0 & 1/2 & 1/2 \end{matrix}$$

is less than

$$\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

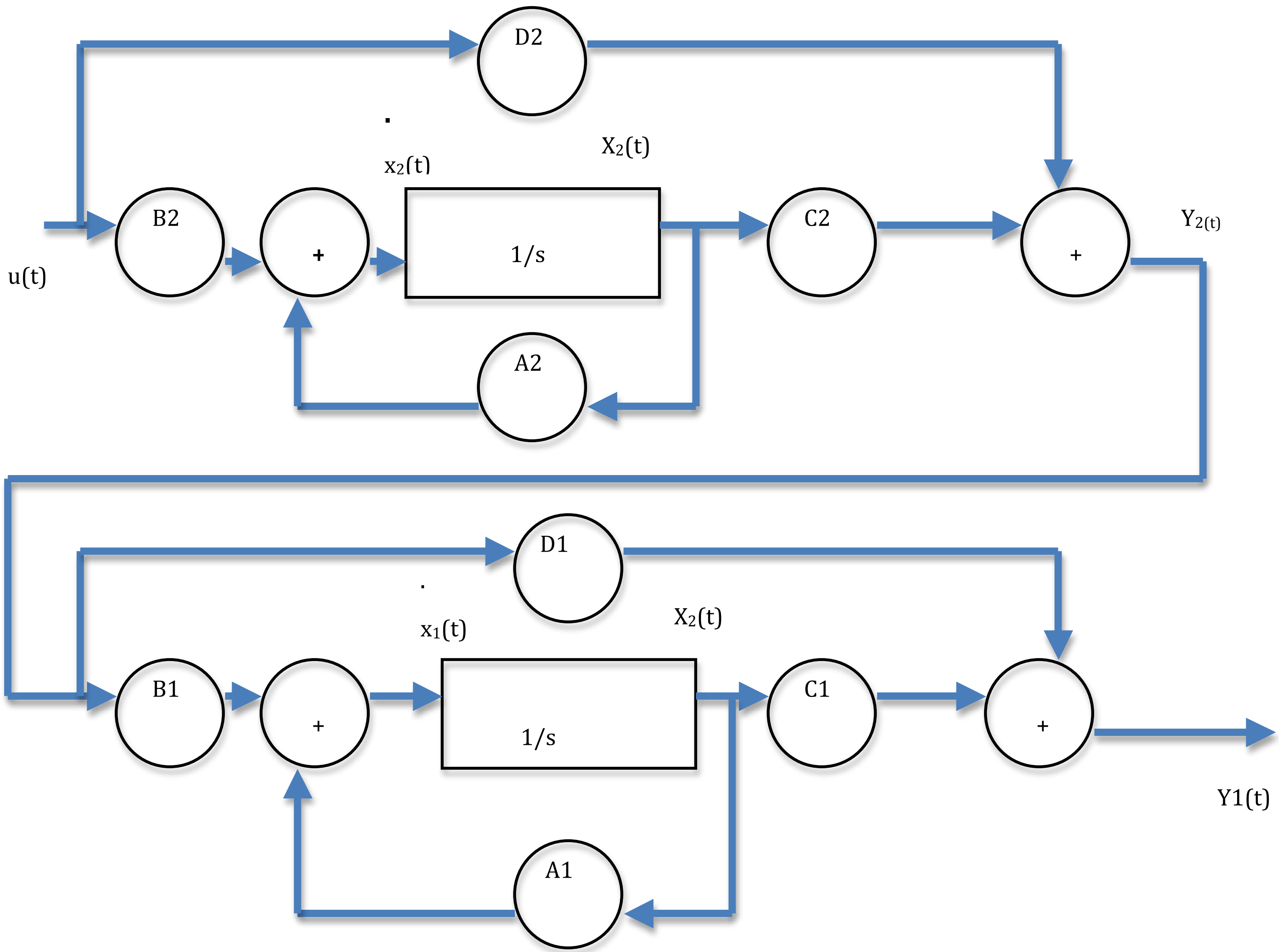
times

$$\begin{matrix} 1 & -1 & -1 & 0 \\ -1 & 1 & -1 & 0 \\ -1 & -1 & 1 & -1 \\ 0 & 0 & -1 & 1 \end{matrix}$$

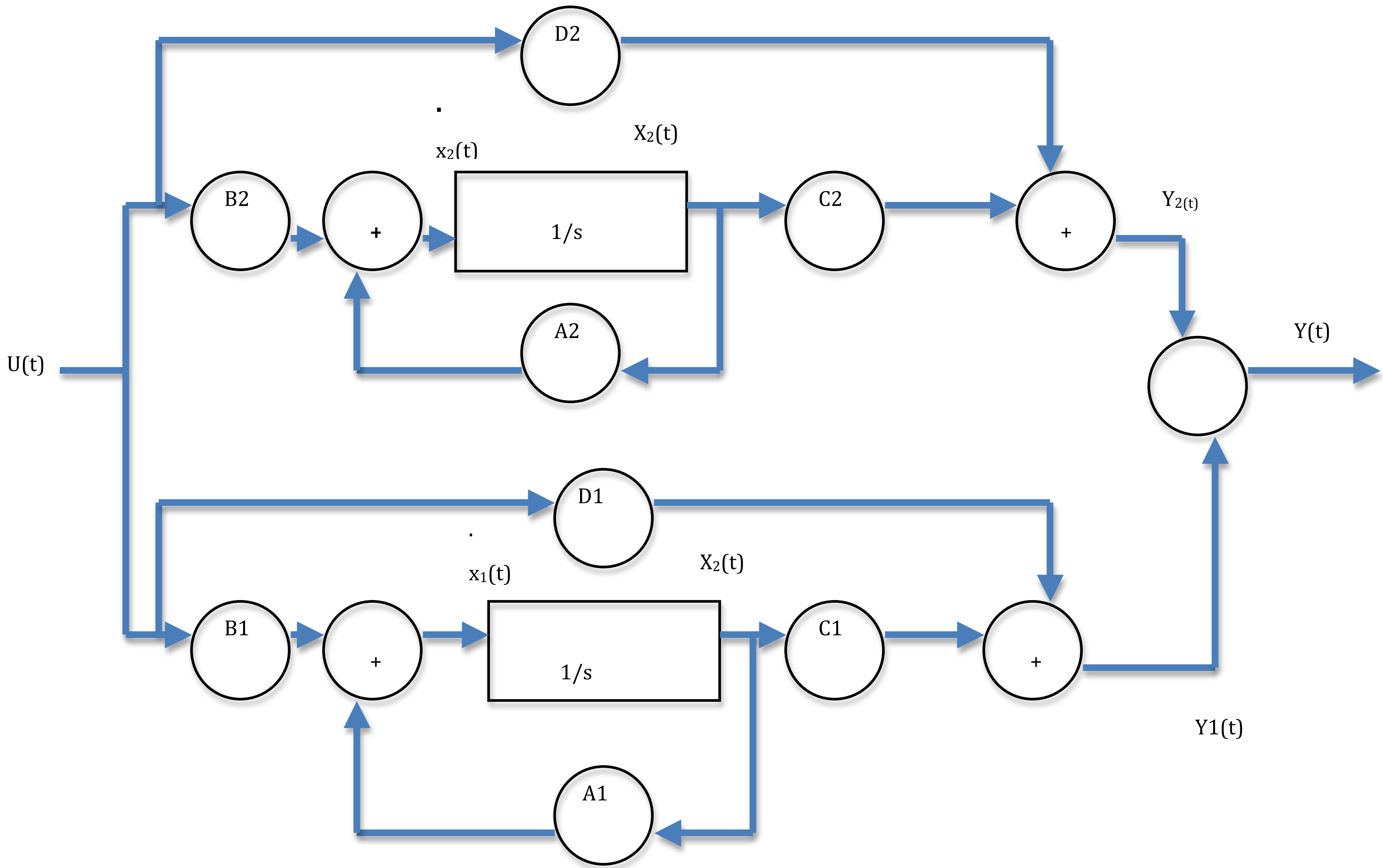
Part I

Using a modified version of theorem previously mentioned in Part G, the matrices mentioned in Part h has met the criteria for achieve asymptotically as k –¿ infinite.

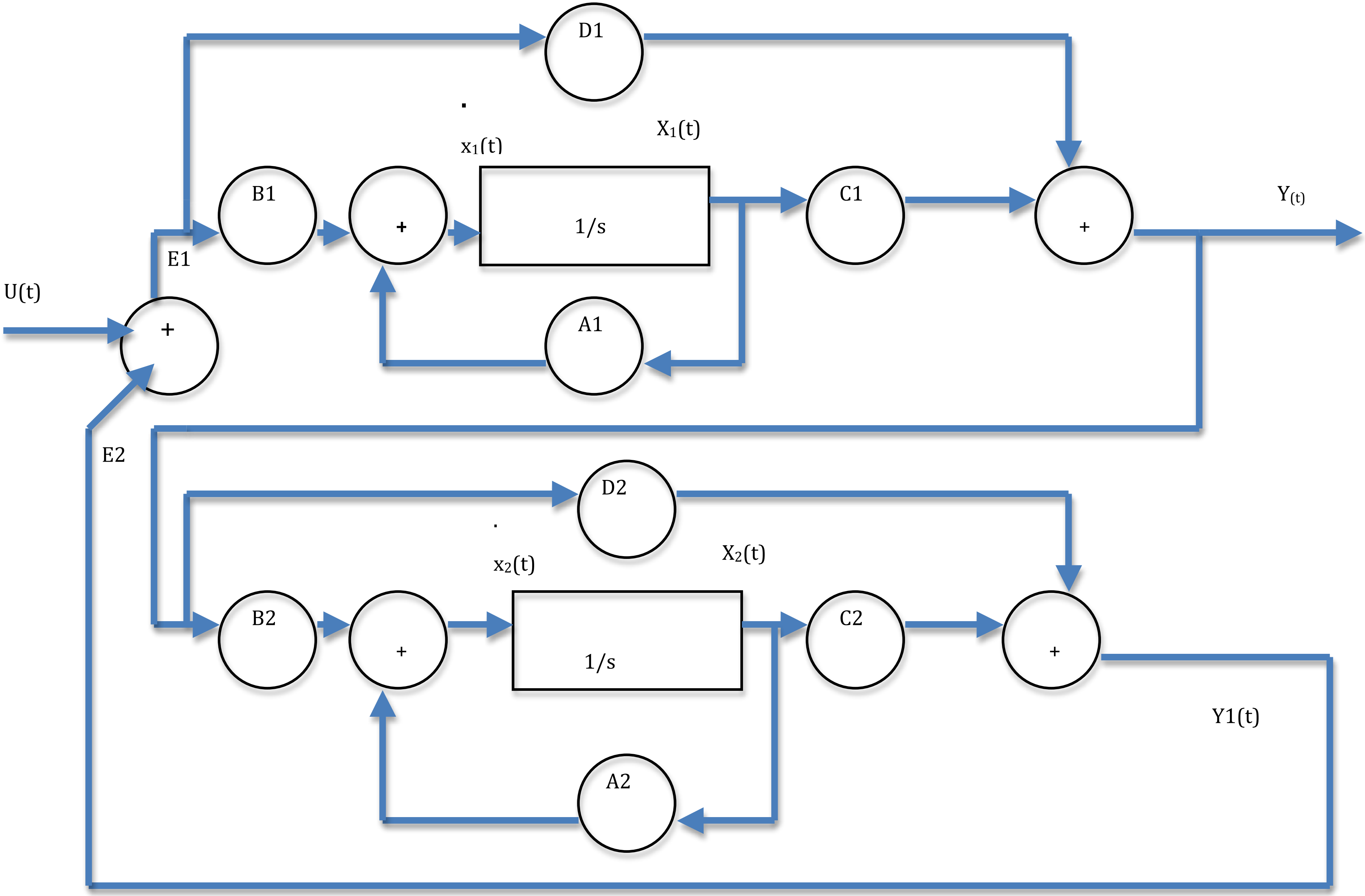
APPENDIX



Problem 1 Part A Block Diagram for Series



Problem 1 Part B Block Diagram for Parellel



Problem 1 Part C Block Diagram for Feedback