EE 301 **Final Exam** **5/3/05**

Name: ____________________

General Instructions:

- Write your name on every page of the exam.
- Do not write on the backs of pages. If you need more paper, it will be provided to you upon request.
- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet will be handed out.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 103 points (a score of 100 or above will be counted as 100).
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 110 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams from everyone.

This exam is for Krogmeier's section of 301.

Do not open the exam until you are told to begin.

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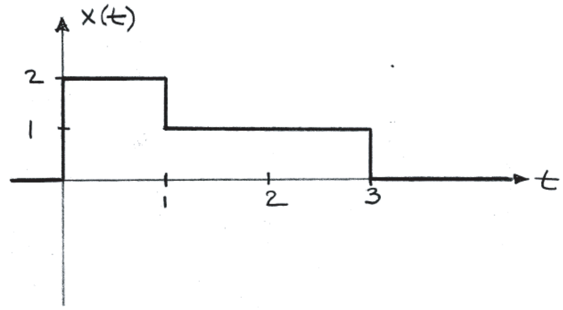
Problem 1. [25 pts. total] Short answer. The following sub-problems may be solved independently.

(a) Consider the pulse $x(t)$ shown below.



(a-i) [3 pts.] Find the energy E_∞ and power P_∞ for the pulse $x(t)$.

(a-ii) [2 pts.] Suppose the pulse is periodically extended to form a new signal $\tilde{x}(t)$ according to:

$$\tilde{x}(t) = \sum_{n=-\infty}^{\infty} x(t - nT).$$

For $T \geq 3$, find the energy $\tilde{E}_\infty$ and power $\tilde{P}_\infty$ of the new pulse in terms of T and your answer to part (a).

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(b) [5 pts.] In class we briefly mentioned that the discrete-time Fourier series is sometimes used to implement linear and time-invariant filtering of discrete-time signals. Suppose that we wish to compute a linear convolution of a signal $x[n]$, which is non-zero for $n = 0, 1, \ldots L-1$, with an impulse response $h[n]$, which is non-zero for $n = 0, 1, \ldots M-1$. We form N-periodic signals

$$\begin{aligned}\tilde{x}[n] &= \sum_l x[n-lN] \\ \tilde{h}[n] &= \sum_l h[n-lN]\end{aligned}$$

where $N \geq \max(L, M)$. Then we let $\tilde{X}_k$ and $\tilde{H}_k$, denote the N-point discrete-time Fourier Series coefficients of $\tilde{x}[n]$ and $\tilde{h}[n]$, respectively. Define

$$\tilde{Y}_k = \tilde{H}_k \tilde{X}_k.$$

What is the minimum N such that the desired linear convolution can be obtained from the first period of $\tilde{y}[n]$, the inverse N-point discrete-time Fourier series of $\tilde{Y}_k$?

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(c) [5 pts.] A signal $x(t)$ has Fourier transform $X(j\omega)$ which is equal to zero for $|\omega| > 10,000\pi$. Suppose the signal undergoes impulse train sampling to create

$$x_p(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t - nT).$$

What is the largest value of T for which $x(t)$ can be recovered exactly from $x_p(t)$?

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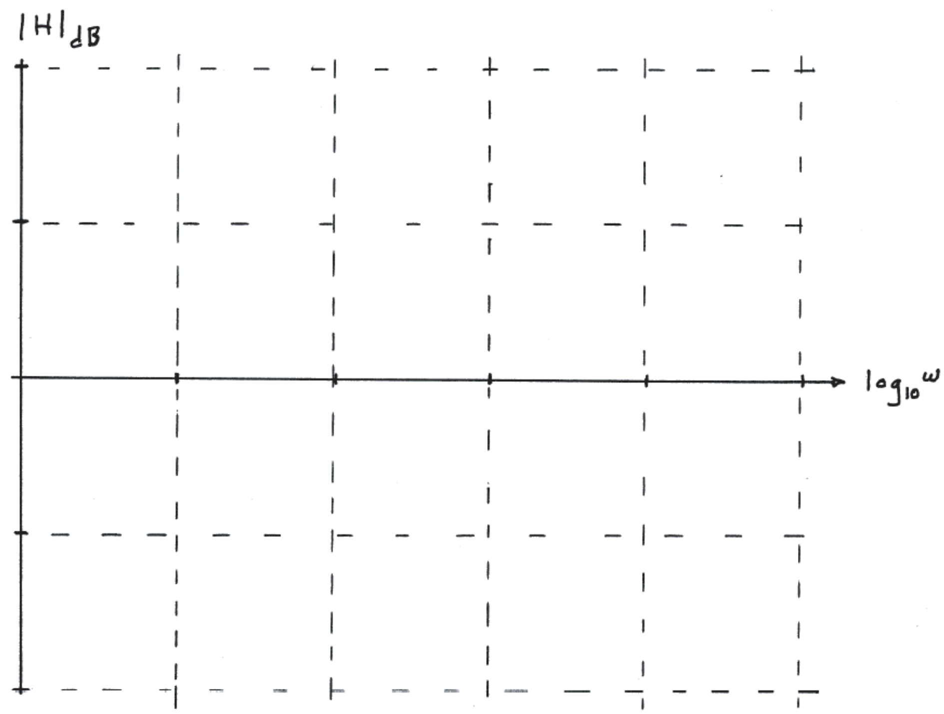
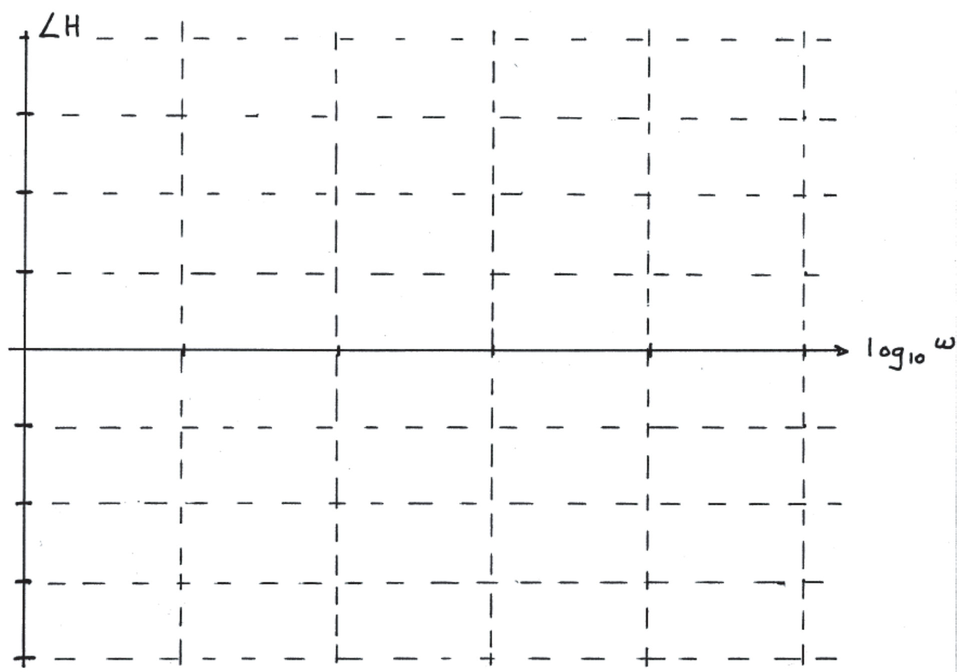
(d) [10 pts.] Plot the Bode magnitude and phase of the following transfer function on the axes given. Carefully label the axes.

$$H(j\omega) = \frac{j\omega}{(1 + j\omega/10)(1 + j\omega/100)(1 + j\omega/1000)}.$$





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Problem 2. [18 pts. total] True or false. For each statement below, indicate if it is true or false. To receive full credit, you must explain your answer.

(a) [2 pts.] T or F: If $f(t) \leftrightarrow F(s)$ is a time signal - bilateral Laplace transform pair, then the continuous time Fourier transform of $f(t)$ is always given by

$$F(s)|_{s=j\omega}\,.$$

(b) [2 pts.] T or F: If a discrete-time LTI system has an impulse response $h[n]$ of finite duration, then the system is stable.

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(c) A system with input $x(t)$ and output $y(t)$ is defined by

$$y(t) = \int_{-\infty}^{\infty} (t-\tau)\tau^2 u(t-\tau)x(\tau)\mathrm{d}\tau$$

where $u(\cdot)$ is the unit step.

(c-i) [2 pts.] T or F: The system is causal.

(c-ii) [2 pts.] T or F: The system is time-invariant.

(c-iii) [2 pts.] T or F: The system is linear.

(c-iv) [2 pts.] T or F: The system is bounded input - bounded output stable.

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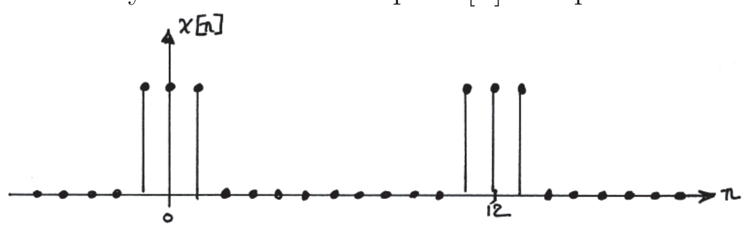
(d) Let $x(t)$ be a periodic signal whose Fourier series coefficients are

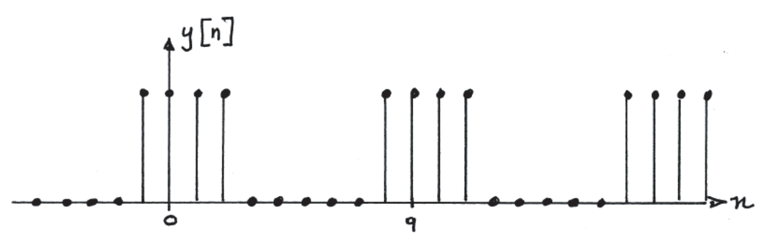
$$x_k = \begin{cases} 2 & k = 0 \\ j(1/2)^{|k|} & \text{otherwise} \end{cases} .$$

(d-i) [2 pts.] T or F: $x(t)$ is real-valued.

(d-ii) [2 pts.] T or F: $x(t)$ is even.

(e) [2 pts.] Let $x[n]$ and $y[n]$ be the periodic discrete-time signals shown below. T or F: There exists an LTI system that takes input $x[n]$ and produces output $y[n]$.





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Problem 3. [20 pts. total] In class we discussed AM large carrier, which allows for the use of a very simple demodulator. We briefly mentioned that the downside of AM large carrier was that its power efficiency was low (AM radio stations broadcast at very high power). The point of this problem is to figure out the power efficiency in a very simple case.

Let

$$y(t) = (A + x(t)) \cos \omega_c t$$

be an AM large carrier signal, where $x(t)$ is the message waveform. Here we only consider a sinusoidal message, i.e.,

$$x(t) = K \cos(\omega_m t + \theta)$$

where $\omega_m \ll \omega_c$.

(a) [3 pts.] Show that the AM large carrier wave may be written as

$$y(t) = A(1 + m \cos(\omega_m t + \theta)) \cos \omega_c t.$$

What is the parameter m and what constraint must be imposed upon it to ensure that overmodulation does not occur?

(b) [5 pts.] By expanding $y(t)$ as a sum of sinusoidal terms and using what you know about the power of a single sinusoid and of sums of sinusoids, find the expression for the total power in $y(t)$.

(c) [3 pts.] From the above expansion of $y(t)$ find the terms that involve the message waveform and then compute the part of the total power that involves the message.

(d) [9 pts.] Find an expression for the power efficiency of AM large carrier for this case of a sinusoidal message by computing the ratio:

$$\eta = \frac{\text{power in message}}{\text{total power}} = \frac{\text{power from part (c)}}{\text{power from part (b)}}.$$

Plot η as a function of m over the allowable range of m from part (a). What is the best possible efficiency η?

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Problem 4. [20 pts. total] Laplace Transform. Parts (a) and (b) are independent of each other.

(a) [10 pts.] Find the (bilateral) Laplace transform, including the region of convergence (ROC), of the signal $x(t) = e^{3t}u(-t+2)$. Do this by a direct calculation from the integral defining the Laplace transform, i.e., do *not* use the transform table or properties directly.

(b) [10 pts.] Find the time function corresponding to the Laplace transform and ROC

$$\frac{s}{s^2+16} \qquad \mathrm{Re}(s) < 0.$$

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Problem 5. [20 pts. total] Z-Transform. Parts (a) and (b) are independent of each other.

(a) [10 pts.] Find the Z-transform of the signal $x[n] = (1/2)^{|n|}$, plot its pole-zero diagram and indicate the ROC.

(b) [10 pts.] Suppose that a time signal $x[n]$ has Z-transform

$$X(z) = \frac{1 - 2z^{-1}}{1 + 2.5z^{-1} + z^{-2}}.$$

Suppose also that $x[n]$ has a DTFT. Find $x[n]$.