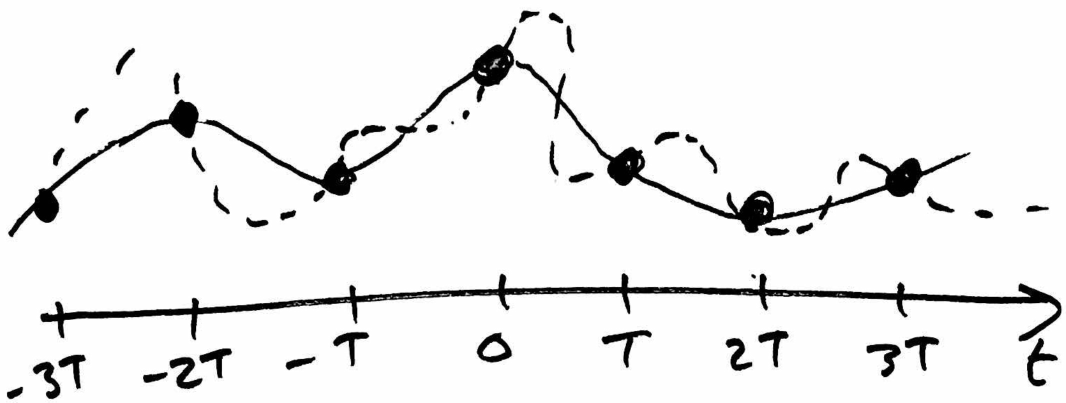
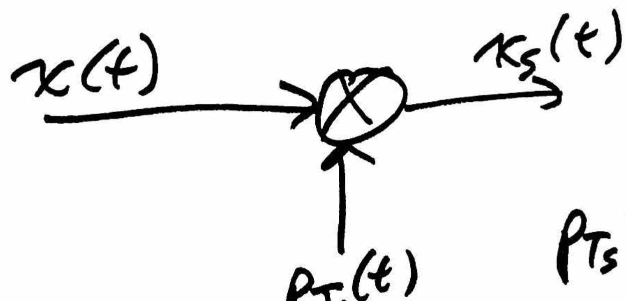
Chap 7 - Sampling

The goal of sampling is to characterize a continuous signal while only having samples of the original signal. Ideally, we would like to sample without loss of information.



Both give the sample points.

To explore this, let's look at an ideal sampler:



$$p_{T_s}(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT_s)$$

$T_s \sim$ sampling period

$$x_s(t) = x(t) \cdot p_{T_s}(t)$$

$$x_s(t) = x(t)\cdot\sum_{k=-\infty}^{\infty}\delta(t-kT_s)$$

$$= \sum_{k=-\infty}^{\infty} x(t)\,\delta(t-kT_s)$$

$$= \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s)$$

From the table:

$$\sum_{k=-\infty}^{\infty}\delta(t-kT_s) \longleftrightarrow \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta\left(\omega-k\frac{2\pi}{T_s}\right) \quad \leftarrow \omega_s = \frac{2\pi}{T_s}$$

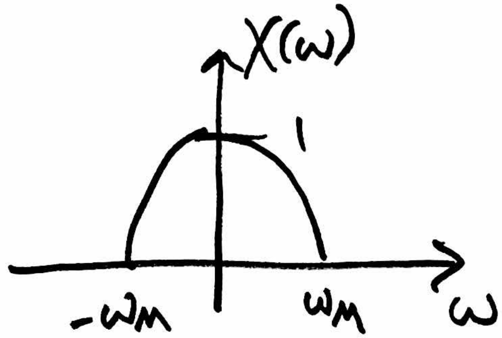
$$= \frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_s)$$

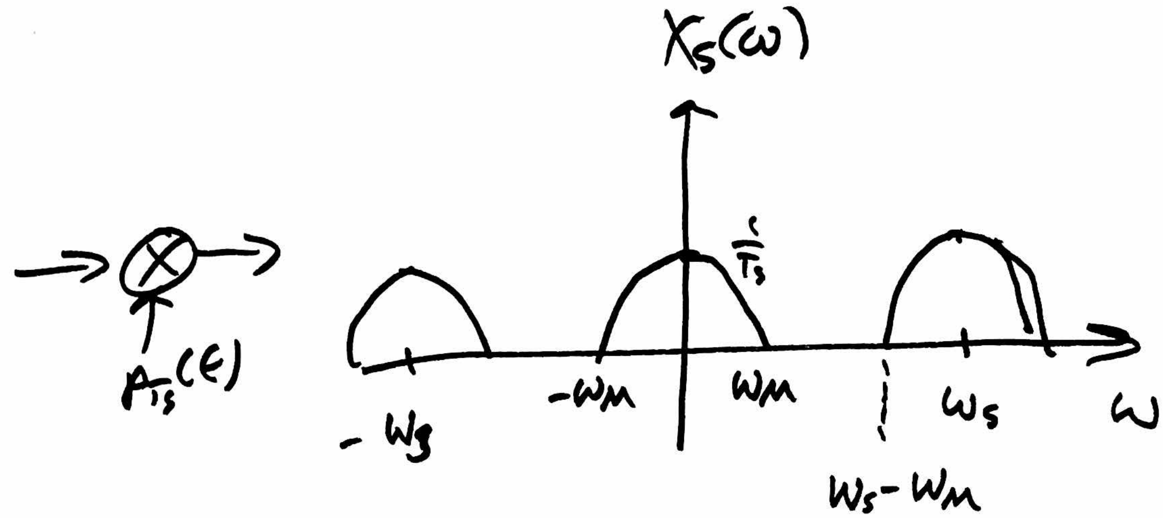
sampling frequency

$$X_s(\omega) = \mathcal{F}\{x(t)p_{T_s}(t)\} = \frac{1}{2\pi}\left(X(\omega) * \left(\frac{2\pi}{T_s}\sum_{k=-\infty}^{\infty}\delta(\omega-k\omega_s)\right)\right)$$

$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(\omega) * \delta(\omega-k\omega_s)$$

$$= \frac{1}{T_s}\sum_{k=-\infty}^{\infty} X(\omega-k\omega_s)$$





ω_M is largest nonzero frequency of $X(\omega)$

To be able to recover $X(\omega)$, you need

$$\omega_M < \omega_s - \omega_M \implies \omega_s > \underbrace{2\omega_M}_{\text{Nyquist rate}}$$

As long as you sample faster than the Nyquist, you can perfectly reconstruct $x(t)$.

What condition did we impose? The signal to be sampled must be bandlimited.

Aliasing

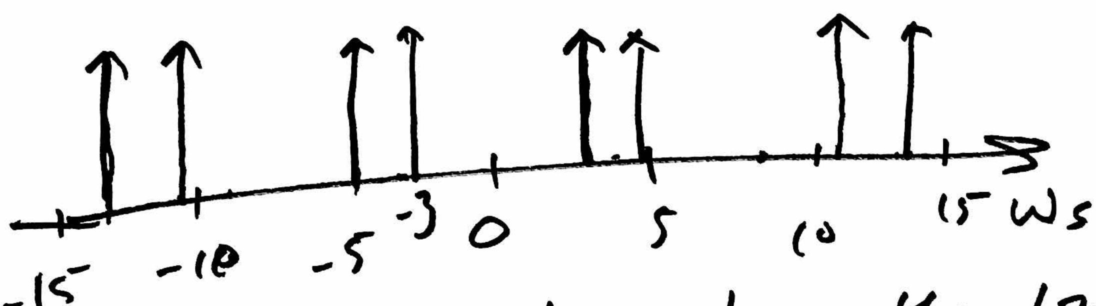
Sampling at too low a frequency (under Nyquist) gives you a representation of the original signal.

Ex

$$x(t) = \cos(5t)$$

$\omega_m = 5$ $\qquad \omega_s > 2 \cdot 5 = 10$

Say we pick $\omega_s = 8$, look at $X_s(\omega)$



If we lowpass filter to capture the lowest frequency copy, we'll get $A(\delta(\omega-3)+\delta(\omega+3))$

$$x_r(t) = B\cos(3t)$$

Ex Exam type system

$$x(t) \longrightarrow \boxed{h_1(t)} \longrightarrow \boxed{h_2(t)} \longrightarrow y(t)$$

$$h_1(t) = \frac{\sin 5t}{\pi t}, \quad h_2(t) = \frac{\sin 3t}{\pi t}$$

Create an equivalent system with the form

$$x(t) \longrightarrow \boxed{h_3(t)} \longrightarrow y(t)$$

$$h_3(t) = h_1(t) * h_2(t) = \mathcal{F}^{-1}\{H_1(\omega) H_2(\omega)\}$$

$$h_3(t) = h_2(t)$$