Spring 2007

(15 pts) **1.** Compute the energy E_∞ and the power P_∞ for the CT signal

$$x(t) = e^{j(2t+\frac{\pi}{4})}.$$

(Justify your answer.)

$|x(t)| = 1$

$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 dt =$

$= \int_{-\infty}^{\infty} dt = t\Big|_{-\infty}^{\infty} = \boxed{\infty}$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} 1 \, dt$

$P_\infty = \lim_{T\to\infty} \frac{1}{2T} 2T = \boxed{1}$

(15 pts) **2.** An LTI system has unit impulse response $h[n] = 3u[n]$. Compute the system's response to the input $x[n] = \left(\frac{1}{2}\right)^{n-1} u[n]$. (Justify your answer.)

$$y[n] = x[n] * h[n]$$

$$\sum_{K=-\infty}^{\infty} \left(\tfrac{1}{2}\right)^{K-1} u[K]\, 3u[n-K]$$

$$K > 0$$

$$\sum_{K=0}^{\infty} \left(\tfrac{1}{2}\right)^{K-1} 3u[n-K]$$

$$n-K > 0 \qquad n > K$$

$$\left\{ \begin{array}{ll} \sum_{K=0}^{n} 3\left(\tfrac{1}{2}\right)^{K-1} & \text{for } n > K \\ 0 & \text{else} \end{array} \right\} = \left\{ \begin{array}{ll} \sum_{K=0}^{n} 3\left(\tfrac{1}{2}\right)^{K}\left(\tfrac{1}{2}\right)^{-1} & \text{for } n > K \\ 0 & \text{else} \end{array} \right\}$$

$$\left\{ \begin{array}{ll} 6\sum_{K=0}^{n} \left(\tfrac{1}{2}\right)^{K} & n > K \\ 0 & \text{else} \end{array} \right\} = \frac{6\left(1-\left(\tfrac{1}{2}\right)^{n+1}\right)}{1-\tfrac{1}{2}} u(n)$$

(20 pts) **3.** Consider the system whose input $x(t)$ is related to the output $y(t)$ by the equation

$$y(t) = x(t-3) + x(3-t).$$

a) Check all properties that that hold for this system. (No justification needed.)

memoryless	
linear	✓
causal	
stable	✓

$y(0) = x(-3) + x(3)$ future

b) Is the above system time invariant? Answer yes/no and justify your answer.

$x(t) \to \boxed{\text{time shift}} \to x(t-t_0) \to \boxed{\text{system}} \to y(t-t_0)$

$= x((t-t_0)-3) + x(3-(t-t_0))$

$x(t) \to \boxed{\text{system}} \to y(t)$

$y(t) = x(t-3) + x(3-t) \to \boxed{\text{time shift}}$

$x(t-3-t_0) + x(3-t-t_0)$

Same is time variant

$x(t-t_0-3) + x(3-t+t_0) \neq x(t-3-t_0) + x(3-t-t_0)$

(15 pts) **4.** Suppose we are given the following information about a signal $x[n]$:

cos wave

1. $x[n]$ is a real and even signal. $f(-t)=f(t)$
2. x[n] has period $N = 10$ and Fourier coefficients a_k.
3. $a_{11} = 5$. $a_1 = 5$
4. $\frac{1}{10}\sum_{n=0}^{9}|x[n]|^2 = 50$. $\sum_{k=0}^{9}|a_k|^2 = 50$

$\frac{2\pi}{x} = 10$

Find $x[n]$. (Justify your answer.) $5^2+5^2=50$

$\frac{\pi}{5} = w_0$

$a_1 = 5$

$a_{-1} = 5$

$$\frac{(2)(5)e^{-jn\frac{\pi}{5}}+(2)(5)e^{jn\frac{\pi}{5}}}{2} = \boxed{10\cos\left(\frac{\pi}{5}n\right)}$$

(15 pts) **5.** Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}de = \frac{sin(4\omega)}{\omega}.$$

If the input to this system is a periodic signal

$$\frac{2\pi}{x} = 8$$

$$x(t) = \begin{cases} 1, & 0 \le t < 4 \\ -1, & 4 \le t < 8 \end{cases} \quad \text{Find } a_K\text{s}$$

$$x = \frac{\pi}{4} = \omega_0$$

with period $T = 8$, determine the corresponding system output $y(t)$.

$$a_K = \frac{1}{T}\int_0^T x(t)e^{-jK(\frac{2\pi}{T})t}dt = \frac{1}{8}\int_0^4 e^{-jK\frac{\pi}{4}t} - \frac{1}{8}\int_4^8 e^{-jK\frac{\pi}{4}t}$$

$$a_K = \frac{1}{8}\left[\frac{e^{-jK\frac{\pi}{4}t}}{-jK\frac{\pi}{4}}\Big|_0^4 - \frac{e^{-jK\frac{\pi}{4}t}}{-jK\frac{\pi}{4}}\Big|_4^8\right]$$

$$a_K = \frac{1}{8}\left[\frac{-e^{-jK\pi}+1}{jK\frac{\pi}{4}} + \frac{\overbrace{e^{-jK2\pi}}^{=1} + e^{-jK\pi}}{jK\frac{\pi}{4}}\right]$$

$$a_K = \frac{2-2e^{-jK\pi}}{2jK\pi} = \frac{1-e^{-jK\pi}}{jK\pi} = \frac{1-(-1)^K}{jK\pi}$$

$a_0 = 0$

$a_1 = a_3 = a_5 = a_7 = \frac{2}{jK\pi}$ $\qquad a_2 = a_4 = a_6 = 0$

$y(t) = a_K H(j\omega)$ $\quad \omega = \frac{\pi}{4}$, so $\frac{\sin(\pi)}{\pi/4} = 0$

$$\boxed{y(t) = 0}$$