ECE 495N EXAM II
Friday, Nov. 6, 2009 (in class) GRIS 280, 1130A-1220P

<u>CLOSED BOOK:</u> The following equations will be provided in the exam.

$h(\vec{k}) = \sum_m [H_{nm}] \exp(i\vec{k}.(\vec{d}_m - \vec{d}_n))$	***Bandstructure***
$D(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))$	***Density of states***
$M(E) = \sum_{\vec{k}} \delta(E - \varepsilon(\vec{k}))\, \pi\hbar v_z(\vec{k})/L$	***Density of modes***
$C_Q = \quad q^2 \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f}{\partial E}\right) D(E)$	***Quantum Capacitance***
$G_B = \quad \frac{q^2}{h} \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f}{\partial E}\right) M(E)$	***Ballistic Conductance***
$f(E) = \frac{1}{e^{(E-\mu)/kT} + 1}$	***Fermi function***

The exam will have three questions (maximum score: 8+8+9 = 25) covering the three topics we have covered since Monday Sept.28 until Friday Oct.30 in class as described in the course outline:

4 / Bandstructure Toy examples, general result, common semiconductors	81-93, 104-116
5 / Subbands Quantum wells, wires, dots and nanotubes	129-137
6 / Density of states and density of modes minimum resistance, quantum versus electrostatic capacitance	138-176

All page numbers refer to the recommended reference #1: ***(QTAT)*** S. Datta, Quantum Transport: Atom to Transistor, Cambridge University Press (2005), ISBN 0-521-63145-9.

Non-MATLAB HW questions are a good guide to the kinds of questions on the test. In addition a few example questions are listed below to help you prepare. Solutions to these will also be posted.

#1. In class we have seen that an infinitely long linear 1-D lattice (lattice constant: a) with a Hamiltonian

$$H = \begin{bmatrix} \varepsilon & t & 0 & 0 & \cdots \\ t & \varepsilon & t & 0 & \cdots \\ 0 & t & \varepsilon & t & \cdots \\ & \cdots & \cdots & \cdots & \end{bmatrix}$$

has a dispersion relation $E(k) = \varepsilon + 2t\cos ka$ where $-\pi < ka < +\pi$ (1)

Do the same problem using a unit cell of ***two*** atoms (instead of one) and show that

$$E(k) = \varepsilon \pm 2t\cos ka \text{ where } -\pi/2 < ka < +\pi/2 \qquad (2)$$

Are (1) and (2) equivalent? Explain.

#2. Suppose electrons have an E(k) relation given by

$$E(\vec{k}) = \sqrt{m^2c^4 + \hbar^2c^2(k_x{}^2 + k_y{}^2 + k_z{}^2)}$$

Obtain an expression for the density of states D(E) in terms of the energy E, the volume V and constants like m, c and $\hbar$.

How would you write the energies of the subbands if the electrons are confined in a thin sheet of thickness "W"?

Obtain an expression for the density of states of one of the subbands.

#3. Problem E.7.5, page 182 from *QTAT*.