key relationship btw DT freq and CT freq :

$$\boxed{\omega_d = \frac{\omega_a}{F_s}}$$

- Sampling on Analog Sinewave

$x_a(t) = e^{j\omega_a t}$, ω_a : analog freq in radians/sec

$F_s = \frac{1}{T_s}$: number of samples/sec (sampling rate in Hz)

$$x[n] = x_a(nT_s) = x_a\left(\frac{n}{F_s}\right)$$

$$= e^{j\omega_a n T_s} = e^{j\omega_d n}$$

where $\underline{\omega_d} = \frac{\omega_a}{F_s} = \omega_a T_s$, n: Integer

$\uparrow$ discrete time freq.

- Consider $x_{a\ell}(t) = e^{j(\omega_a + \ell 2\pi F_s)\underline{t}}$, ℓ : integer

$$x_\ell[n] = x_{a\ell}(\underline{nT_s}) = x_{a\ell}\left(\frac{n}{F_s}\right)$$

$$= \underline{e^{j\omega_a n T_s}}\, e^{j\ell 2\pi \cancel{F_s} n \cancel{T_s}}$$

$$= e^{j\omega_d n}\, \cancel{e^{j\ell 2\pi n}}_{=1}$$

$$= e^{j\omega_d n} \quad , \quad \omega_d = \frac{\omega_a}{F_s}$$

$e^{j\omega_a t} \xrightarrow{\text{sample}} e^{j\omega_d n}$

$e^{j(\omega_a + \ell 2\pi F_s)t} \Rightarrow e^{j\omega_d n}$

↳ any two analog freq seperated by $\ell\omega_s$ $(= \ell 2\pi F_s)$ get mapped to same DT sinewave

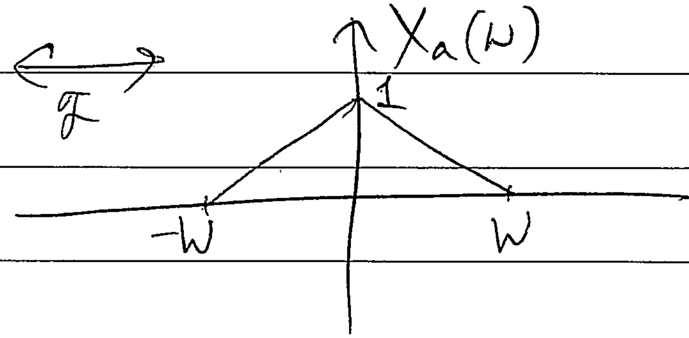
↳ Since $\omega_d = \frac{\omega_a}{F_s}$, any 2 CT sinewaves seperated by $\underline{\ell\omega_s}$ yields 2 DT sinewaves seperated by $\underline{\ell 2\pi}$ (same DT sinewave)

$$\frac{\ell\omega_s}{F_s} = \frac{\ell 2\pi \cancel{F_s}}{\cancel{F_s}} = \ell 2\pi$$

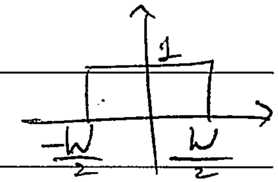
Example $\quad x_a(t) = \dfrac{2\pi}{W}\left\{\dfrac{\sin\left(\frac{W}{2}t\right)}{\pi t}\right\}^2$

sampled at a rate $\omega_s > 2\omega_{max}$, where $\omega_{max} = W$

① $\xleftrightarrow{\mathcal{F}}$

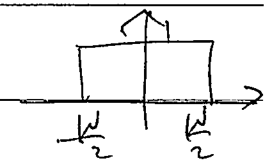


" $\dfrac{\sin\frac{W}{2}t}{\pi t} \xleftrightarrow{\mathcal{F}}$



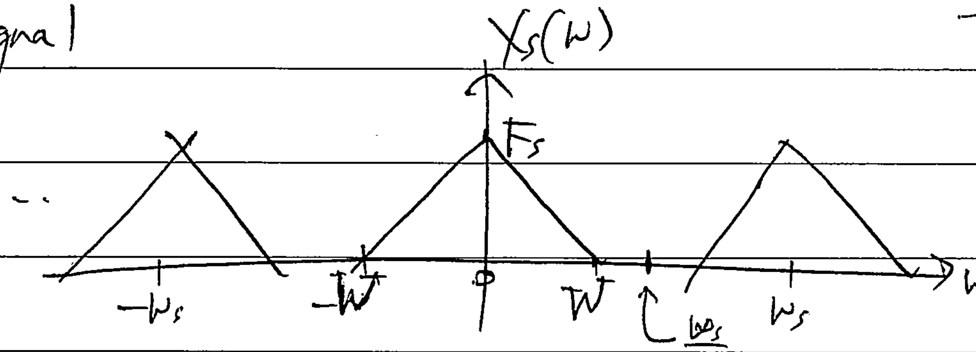
"

$*$ $\quad \times\frac{1}{2\pi}$

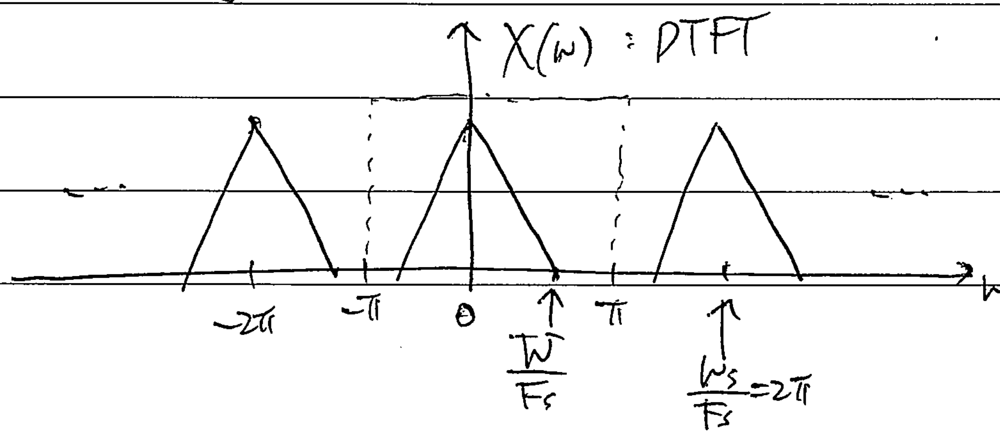


② For sampled signal

$x_s(t) \xleftrightarrow{\mathcal{F}}$



③ Compress / divide by sampling rate $F_s = \frac{1}{T_s}$



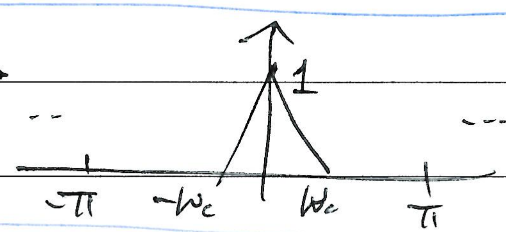
Note: $\circ$ $x[n] = x_a(nT_s) = \frac{2\pi}{W}\left\{\frac{\sin\left(\frac{W}{2}nT_s\right)}{\pi n T_s}\right\}^2$

$$= \frac{2\pi}{W}\left\{\frac{\sin\left(\frac{W}{2F_s}n\right)}{\pi n / F_s}\right\}^2 = \frac{2\pi F_s^2}{W}\left\{\frac{\sin\left(\frac{w_c}{2}n\right)}{\pi n}\right\}^2$$

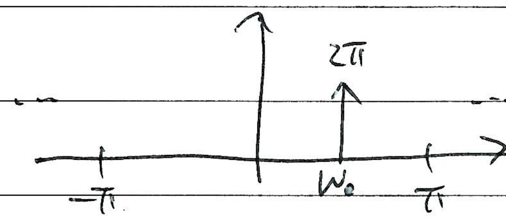
where : $w_c = \frac{W}{F_s}$

$\circ$ Now divide by F_s $\Rightarrow$ divide by F_s in freq domain and we get our first DTFT pair

$$\boxed{\frac{2\pi}{w_c}\left\{\frac{\sin\left(\frac{w_c}{2}n\right)}{\pi n}\right\}^2 \xleftrightarrow{\text{DTFT}}}$$



$$x[n] = e^{jw_0 n} \xleftrightarrow{\text{DTFT}}$$



DTFT : Properties & Examples.

- DTFT $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$
- I-DTFT $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$

* Two Primary differences btw CTFT (Chap. 4) and DTFT (Chap. 5)

- In DT domain, n is a discrete variable
- DTFT is periodic with period 2π

↳ As a result, some of the properties of the DTFT are quite different than the properties of the CTFT, although some are similar.

- Similar properties $\quad x[n] \overset{DTFT}{\longleftrightarrow} X(\omega)$
  - time-shift $\quad x[n-n_0] \overset{DTFT}{\longleftrightarrow} e^{-j\omega n_0} X(\omega)$
  - convolution $\quad y[n] = x[n] * h[n] \overset{DTFT}{\longleftrightarrow} Y(\omega) = H(\omega) X(\omega)$
  - modulation $\quad e^{j\omega_0 n} x[n] \overset{DTFT}{\longleftrightarrow} X(\omega - \omega_0)$

    but need to shift all periods of $X(\omega)$ to the right by ω_0 (example shortly)

- Different properties.
  - since n is discrete while ω is continuous, there is no duality property for DTFT.
  - since n must be an integer, the time-scaling or freq-scaling property is very different.

(recall) $x(at) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$, true for any a.

CTFT

For DTFT, we have two corresponding properties.

① $y[n] = x[Dn] \overset{DTFT}{\longleftrightarrow} Y(\omega) = \frac{1}{D}\sum_{k=0}^{D-1} X\left(\frac{\omega - k2\pi}{D}\right)$

, where D : integer > 1

③ $y[n] = \begin{cases} x\left[\frac{n}{L}\right] & , n = lL \\ 0 & , o/w \end{cases} \overset{DTFT}{\longleftrightarrow} Y(\omega) = X(L\omega)$

, where L, l : integer

↳ These two prop. are beyond the scope of 301.

so you will not be tested on them. //

(time-d) (freq-d)

conti- signal $\xrightarrow{CTFT}$ aperiodic

discrete signal $\xrightarrow{DTFT}$ periodic