Chapter 3

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3:43 PM

complex exponentials are "eigenfunctions" of LTI systems.

$e^{zt},\ z \in \mathbb{C} \rightarrow \boxed{LTI} \rightarrow \underset{\text{number}}{H(z)}\, e^{z(t)}$

eigen functions

eigen value

depends on z

$z^n \ \boxed{LTI} \rightarrow H(z) z^n$

e.g. $(1+2j)^n \rightarrow H(1+2j)\ (1+2j)^n$

ex. $x(t) = e^t$

$e^t e^{-\tau}$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, e^{t-\tau}\, d\tau = e^t \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-\tau}\, d\tau}_{\substack{\text{number} \\ h(z)}}$$

so $e^t \rightarrow \boxed{sys} \rightarrow \text{constant} \cdot e^t$

ex. $x(t) = e^{zt}$

$e^{zt} e^{-zt}$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau)\, e^{z(t-\tau)} =$$

$$= e^{zt} \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-z\tau}\, d\tau}_{\text{number not dependent on } t} = x(t) \cdot \text{number}$$

So

it's a good idea to write your input as a linear combination of complex exponentials.

E.g. $x(t) = 7e^{jt} + 8e^{-t} + 9e^{3t}$

output would be $y(t) = 7H(j)e^{jt} + 8H(-1)e^{-t} + 9H(3)e^{3t}$

$\Rightarrow$ Fourier series
Fourier Transforms

Fourier series

Let $x(t)$ be a periodic CT signal
Let T be the fundamental period $x(t)$
write $\omega_0 = \frac{2\pi}{T}$ Then $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$, where

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\omega_0 t}\,dt$$

This series is called the Fourier series of $x(t)$
a_k's are called Fourier coefficients

Q Compute fourier series of $x(t) = 3\cos 3t + (1+6j)\sin 6t$

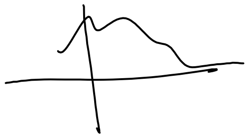
recall $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$ $\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

$$x(t) = 3\left(\frac{e^{j3t} + e^{-j3t}}{2}\right) + (1+6j)\left(\frac{e^{j6t} - e^{-j6t}}{2}\right)$$

$$= \underbrace{\frac{3}{2}}_{a_1} \underbrace{e^{j3t}}_{k=1} + \underbrace{\frac{3}{2}}_{a_1} \underbrace{e^{-j3t}}_{k=1} + \underbrace{\frac{(1+6j)}{2}}_{a_2} \underbrace{e^{j6t}}_{k=2} - \underbrace{\left(\frac{1+6j}{2}\right)}_{a_2} \underbrace{e^{-j6t}}_{k=2}$$

W_0 - fundamental frequency $= 3$ $T_0 = \frac{2\pi}{3}$

average of signal over the period

So  $\Rightarrow$ $(\ldots a_{-3}, a_{-2}, a_{-1}, a_0, a_1, a_2, a_3)$