1. Consider the following CT signals:

$$\begin{aligned} x(t) &= \frac{\sin(5\pi t)}{\pi t}, \\ y(t) &= x(t)e^{j\omega_1 t}, \\ z(t) &= x(t)\cos(\omega_2 t). \end{aligned}$$

6y(19)

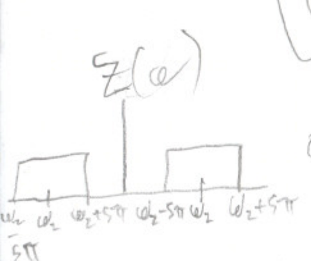
(5 pts) a) Is the signal $x(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?

Yes. $X(\omega) = u(\omega + 5\pi) - u(\omega - 5\pi)$, which is 0 for $|\omega| > 5\pi$. Thus, the Nyquist rate is $2 \cdot 5\pi = \boxed{10\pi = \omega_N}$

(5 pts) b) Assuming that $\omega_1 > 0$, is the signal $y(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?

Yes. By (12), $Y(\omega) = X(\omega - \omega_1)$. This is the same as $X(\omega)$, recentered on ω_1 (see figure at left). The maximum frequency magnitude is then $|\omega_1 + 5\pi| = \omega_M$. The Nyquist rate $\boxed{\omega_N = 2\omega_M = 2\omega_1 + 10\pi}$

(5 pts) c) Assuming that $\omega_2 > 0$, is the signal $z(t)$ band limited? (Answer yes/no and justify.) If you answered yes, what is the signal's Nyquist rate?



Yes. Since $\cos(\omega_2 t) = \frac{1}{2}(e^{j\omega_2 t} + e^{-j\omega_2 t})$, $z(t) = x(t)\cos(\omega_2 t)$ produces two copies of $X(\omega)$ at $+\omega_2$ and $-\omega_2$. $Z(\omega) = \frac{1}{2}(X(\omega - \omega_2) + X(\omega + \omega_2))$ by (12). The Nyquist rate is then $\boxed{\omega_N = 2(\omega_2 + 5\pi)}$ as can be seen in the figure at left.

(10 pts) d) Can one recover x(t) from y(t)? (Answer yes/no. If you answer yes, explain how. If you answered no, explain why not.)

$\boxed{\text{Yes}}$. Simply recover x(t) as:

$x_r(t) = y(t)e^{-j\omega_1 t} = x(t)e^{j\omega_1 t}e^{-j\omega_1 t} = x(t)$

You simply multiply y(t) by $e^{-j\omega_1 t}$.

(15 pts) e) Can one recover x(t) from z(t)? (Answer yes/no. If you answer yes, explain how. If you answered no, explain why not.)

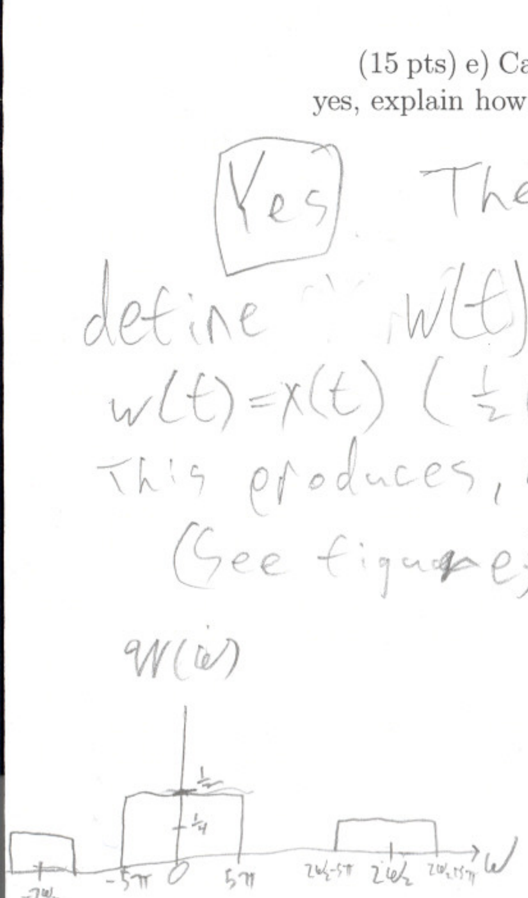
$\boxed{\text{Yes}}$. The method given in class was to define $w(t) = z(t)\cos\omega_2 t = x(t)\cos^2\omega_2 t$.

$w(t) = x(t)\left(\frac{1}{2}\left(e^{j\omega_2 t} + e^{-j\omega_2 t}\right)\right)^2 = x(t)\left(\frac{1}{4}\left(e^{j2\omega_2 t} + 2 + e^{-j2\omega_2 t}\right)\right)$

This produces, by (12), $W(\omega) = \frac{1}{4}X(\omega - 2\omega_2) + \frac{1}{2}X(\omega) + \frac{1}{4}X(\omega + 2\omega_2)$

(See figure e).

$W(\omega)$



You then apply a lowpass filter with gain 2 to $W(\omega)$, making sure the cutoff ω_c is such that $5\pi < \omega_c < 2\omega_2 - 5\pi$. The lowpass filter output is x(t).

(15 pts) f) Impulse-train sampling is used to obtain

$n - kN = 0$

$kN = n$

$k = \frac{n}{N}$

$$x_p[n] = \sum_{k=-\infty}^{\infty} x(n)\delta(n - kN).$$

If the sampling period is $N = \frac{2}{11}$, will aliasing occur? Justify your answer.

The Nyquist rate $\omega_N = 10\pi$ for $x(t)$. In order to avoid aliasing the sampling rate ω_s must be such that $\omega_s > \omega_N$. $\omega_s = \frac{2\pi}{N} = \frac{2\pi}{\frac{2}{11}} = 11\pi = \omega_s$.

The condition $\omega_s > \omega_N$ is satisfied, so there is $\boxed{\text{no aliasing}}$.

(15 pts) g) Impulse-train sampling is used to obtain

$$y_p[n] = \sum_{k=-\infty}^{\infty} y(n)\delta(n - kN).$$

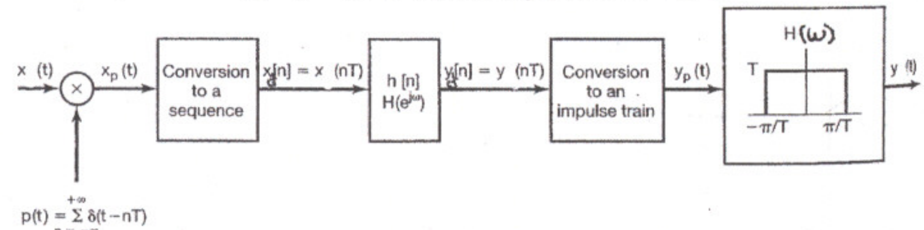
If $\omega_1 = -3\pi$ and the sampling period is $N = \frac{1}{6}$, will aliasing occur? Justify your answer.

The Nyquist rate ω_N of $y(t)$ is $2|\omega_1 + 5\pi|$ as indicated in 1.b. $\omega_N = 2|-3\pi + 5\pi| = 4\pi$

The sampling rate $\omega_s = \frac{2\pi}{N} = 6 \cdot 2\pi = 12\pi$.

$\omega_s > \omega_N \longrightarrow$ $\boxed{\text{no aliasing}}$

(20 pts) **2.** The following figure shows the overall system for filtering a CT signal using a DT filter. If $\mathcal{X}(\omega)$ and $\mathcal{H}_d(\omega)$ are as shown below, with $\frac{1}{T} = 20\text{kHZ}$, sketch $\mathcal{X}_p(\omega)$, $\mathcal{X}_d(\omega)$, $\mathcal{Y}_d(\omega)$, $\mathcal{Y}_p(\omega)$, and $\mathcal{Y}(\omega)$.



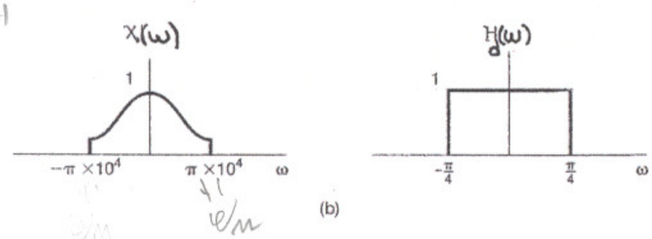
(a)

$\omega_s = 2\pi \cdot \frac{1}{T} = 4\pi \cdot 10^4$

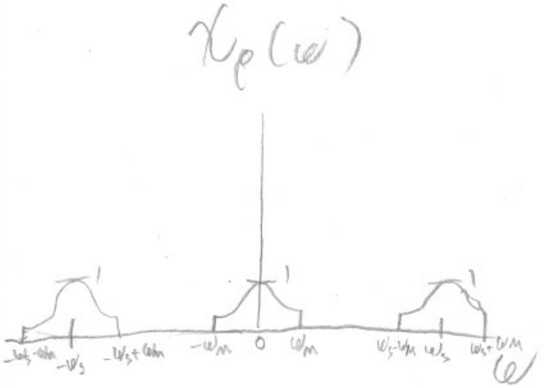
$\omega_M = \pi \cdot 10^4$

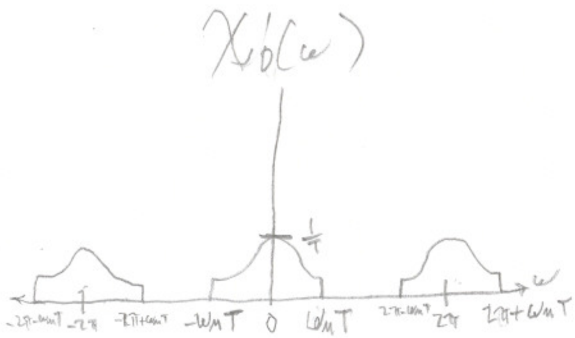
$\omega_M T = \frac{\omega_M}{1/T} = \frac{\pi \cdot 10^4}{2 \cdot 10^4}$

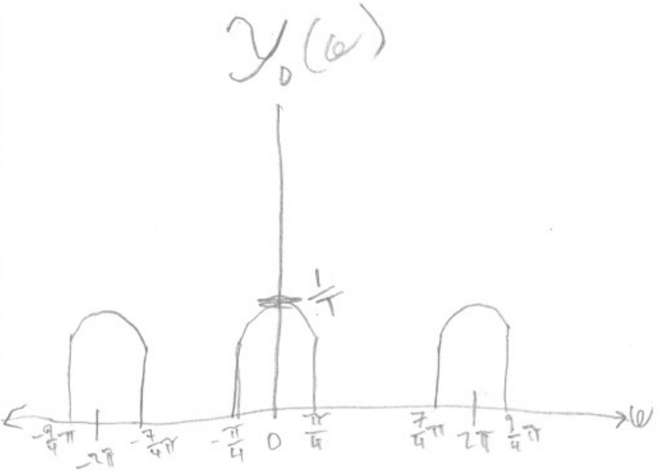
$\omega_M T = \frac{\pi}{2}$

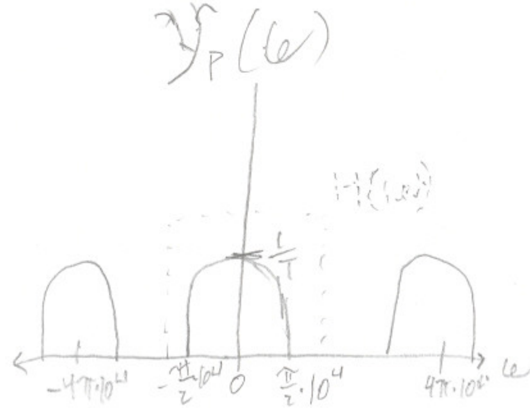


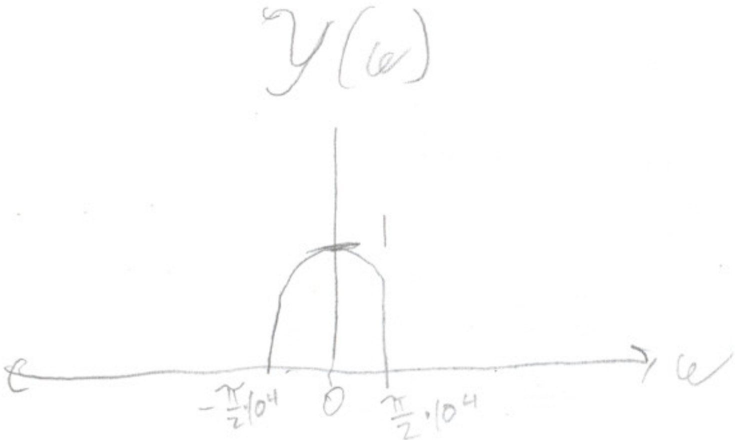
(b)











$Y(\omega)$
1
ω
$-\frac{\pi}{2} \cdot 10^4$
0
$\frac{\pi}{2} \cdot 10^4$

(15 pts) **3.** Using the definition of the Laplace transform (i.e. do not simply take the answer from the table), compute the Laplace transform of

$$x(t) = e^{-5t}u(-t)$$

$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{\infty} e^{-5t} u(-t)\, e^{-st}\, dt$$

$$= \int_{-\infty}^{0} e^{-(5+s)t}\, dt$$

$$= \frac{-1}{5+s} e^{-(5+s)t} \Big|_{-\infty}^{0}$$

$-(5+s)\cdot -\infty$

$(5+s)\infty$

$$= \frac{-1}{5+s} - 0, \text{ IF } \operatorname{Re}(s) + 5 < 0, \text{ else diverges}$$

$$\boxed{X(s) = \frac{-1}{5+s}, \quad \text{ROC: } \operatorname{Re}(s) < -5}$$