Experiment 2: Linear and Non-Linear Systems

I. OBJECTIVES

Upon completion of this experiment, you should be able to:

1. Characterize linear, time-invariant systems in the time and frequency domains using signal generators and oscilloscopes.
2. Build a simple scalar network analyzer to measure the frequency response of a low-frequency LTI system.
3. Measure the inter-modulation distortion of a simple non-linear system.

II. INTRODUCTION

Frequently you will need to characterize linear systems in terms of their time-domain responses (impulse response, step response) or their frequency domain response. In addition, most 'linear' systems are not perfectly linear; they exhibit weak non-linearities due to manufacturing, environmental, and other effects within their constituent components. These non-linearities can have a profound impact on communication systems. Therefore, it is important to study and characterize them in terms of their 'inter-modulation' products.

III. PRELAB

1. Define impulse response and frequency response for linear time-invariant (LTI) systems.

2 The frequency response of an LTI system is given by $|H(f)|e^{j2\pi\theta(f)}$.
The input to this system is given by $x(t) = 2\cos(2\pi 3000t) + 3\cos(2\pi 4700t)$.
What is the output?

3. Consider a non-linear system defined by $y(t) = x(t) + ax(t)^2 + bx(t)^3$. (Where, a and b are constants, $x(t)$ is the input to the system, and $y(t)$ is the output.)

   If $x(t)$ is given by $x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$, determine $y(t)$ and its spectrum, $Y(f)$. Also determine $X(f)$, the spectrum of $x(t)$.
   (Refer to [1, Appendix G.2, G.3] for a useful list of formulae. Alternatively, the **MATLAB** command ***simple*** can be used to simplify expressions.)

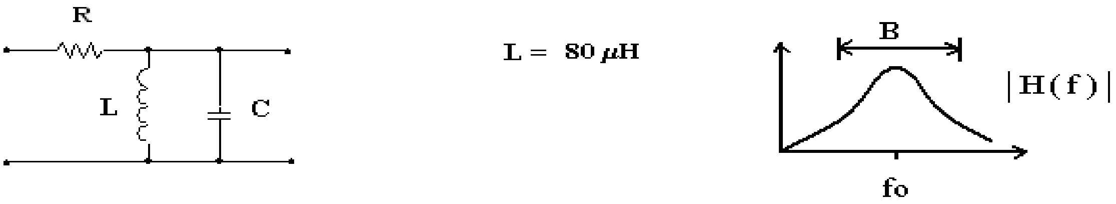
4. When a LTI system is driven by a pure sinusoidal input, the output is also a pure sinusoid at the same frequency. Is this true for linear time-variant systems? Justify.
   Is this true for non-linear systems? Justify.

IV. EXPERIMENT

The goals of this experiment are to characterize an LTI system in the time and frequency domains and to characterize a weakly non-linear system in terms of its inter-modulation distortion (IMD).

1. Measuring the Frequency Response of an LTI System.

1a. Build the RLC band-pass filter (shown in Figure 1.)



$$|H(f)| = K\left[1+\left(\frac{f - fo}{\frac{1}{2}B}\right)^2\right]^{-1/2} \qquad B = \frac{1}{2\pi RC} \qquad fo = \frac{1}{2\pi\sqrt{LC}}$$

Fig. 1: RLC band-pass circuit.

Select C for fo = 400-500 kHz. Select R so that the 3-dB bandwidth, B, is 100 kHz. Determine the step response of this circuit (hint: a step input can be simulated through the use of a sufficiently low-frequency square wave) observe the output of the system using an **oscilloscope**.

1b. Manually plot the frequency response of the band-pass filter
Generate sinusoids at frequencies in the 0-800kHz range and use them as inputs to the BPF. Observe and record the output of the BPF on an **oscilloscope**.
Determine the maximum gain, and the 3-dB cutoff frequencies.
Record Vin(f), Vout(f) and |H(f)| = | Vout(f) / Vin(f) |.
Focus on |H(f)|, the amplitude of the frequency response; ignore the phase function.

1c. Build a simple network analyzer.

Automatic Frequency Scanning.
In 1 b, above, the input frequency to the filter was manually scanned.
Frequency scanning may be automated by using a voltage-controlled oscillator (VCO). The output frequency of a VCO depends on its input voltage.

You will be using the 3314A as a VCO. The characteristics of the 3314A VCO are:

1. The frequency changes roughly 10% of the displayed nominal frequency per volt of input.
2. The input voltage must lie between -10 volts and +1 volt.

For example, if the displayed frequency is 50 kHz, and the input is +0.5 volts, the output frequency is 52.5 kHz (50 + 0.5*10% * 50).

The frequency range must be scanned very slowly to get sensible results.

A rule of thumb is:

The product of sweep-time and frequency span must be much larger than 1.
A signal that satisfies this rule is a wide-band frequency-modulated signal.

Generate a symmetric triangular wave (with as much DC offset as required) using the 33220A and use it as the input (between -10 and +1 volts) to the 3314A VCO. Connect the VCO output to the input of the BPF. Connect the triangular wave as X-input to the oscilloscope, the output of the BPF as Y-input and put the scope in X-Y mode. Measure and record the bandwidth and maximum gain of the BPF from part 1b using your simple low-frequency **scalar network analyzer.**[1] Record 3dB bandwidth and maximum gain; compare to the results from 1b.

2. Measuring the Inter-modulation Distortion of a "Linear" system.

Many 'linear' systems show small amounts of non-linearity especially when driven by large input signals. This is true of many low-noise amplifiers, power amplifiers and filters commonly used in communication systems. Non-linearity results in undesirable frequency components such as inter-modulation products and harmonics. In the following section, the non-linearities of a band-pass filter will be investigated.

2a. The two-tone test.

Perform the two tone test on the **455 kHz BPF** at your station. This filter-amplifier combination has an 18 kHz bandwidth centered at 455 kHz. This unit displays weak non-linearities that produce a measurable inter-modulation distortion for a 4 volt peak-to-peak, two-tone input signal.

The two-tone test is performed as follows:

1. Generate two sinusoids of **equal** amplitude using separate function generators. Pick their frequencies as 450 kHz and 460 kHz. Begin with 1 volt amplitudes.
2. Add these sinusoids and use the resultant "two-tone" signal as input to the BPF.
3. Analyze the BPF output signal on the spectrum analyzer using a logarithmic scale. (The inputs to the analyzer are fragile; pay close attention to the note below)

NOTE: The spectrum analyzer cannot handle large input voltages..

- Always use a **40 dB attenuator** between the signal and the SA input.
- The **40 dB attenuator** is just a 4950Ω resistor.
- This resistor and the 50Ω input resistance of the SA form a 100:1 voltage divider.

NOTE: If at any point, the SA displays a '**Caution: Overload on Input**' sign, disconnect the signal at the SA input and reduce all amplitudes.

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[1] The 4395A can be used as a **vector network analyzer**; "vector" refers to the fact that it is capable of providing both phase and amplitude information. The proper use of the network analyzer requires an understanding of calibration procedures. We will not be doing this in the lab. You can read more about network analyzers in [2], [3].

Record the amplitudes of the fundamentals, harmonics, and intermodulation terms:

fundamentals: $f_1 = 450$ kHz and $f_2 = 460$ kHz
second harmonics: $2f_1 = 900$ kHz and $2f_2 = 920$ kHz
third harmonics: $3f_1 = 1350$ kHz and $3f_2 = 1380$ kHz

inter-modulation terms:

second-order $|f_1 \pm f_2|$: 10 kHz and 910 kHz
third order $|2f1 \pm f2|$, $|f1 \pm 2f2|$: 440 kHz, 470 kHz, 1360 kHz, 1370 kHz

2b. Effect of reduced amplitude on the 3rd order in-band inter-modulation distortion

To reduce the amplitude of the input signal while keeping the two tones equal in power, insert the HP350D Attenuator (see Appendix) between the adder and the BPF.

Observe the effect of reducing the input amplitude.
Record the amplitude and RMS voltage of the corresponding spectral components.
Gather data spaced over **two decades** of input magnitude. (Note: one decade is equal to one order of magnitude)

Measure the fundamentals (450 kHz and 460 kHz) and the in-band third order inter-modulation terms (440 kHz and 470 kHz).

Plot the RMS voltages of the in-band third order terms vs. the amplitude of either fundamental. **Use a logarithmic scale for both axes**.
Discuss your results with respect to Prelab Question 3.

2c. Why are the **in-band** third-order inter-modulation terms especially important?

V. REPORT

Document all the readings you have obtained and any conclusions you draw in your report. Attach a copy of your lab record to the report. Answer any specific questions asked in the lab manual.

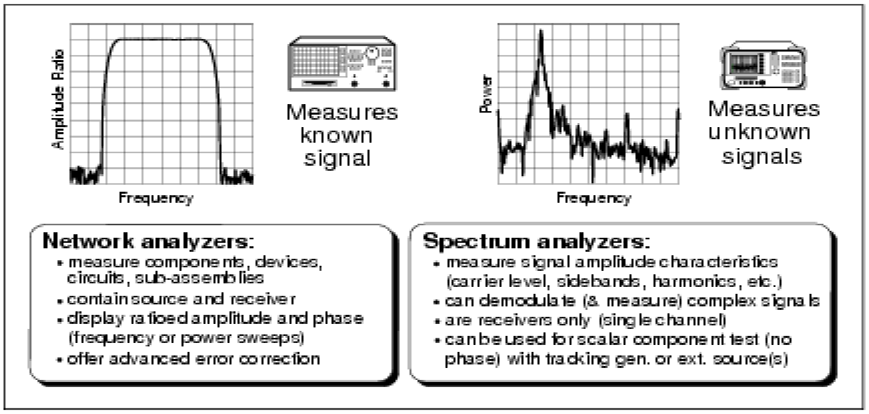
VI. APPENDIX

HP350D Attenuator

The HP350D is a passive resistive-bridge attenuator. For the HP350D attenuator to work roughly as calibrated, it is necessary that the input and output resistances be 600Ω. For example, if the input is driven directly by a 50Ω signal generator like the 33220A and the output resistance is 50Ω, the attenuation will not be as marked. At the very least, one of the two terminations must be 600Ω.

Network Analyzers

A real linear system is not characterized by one frequency (or time) response function. The linear systems you will encounter in the lab are ‘two-ports’; four response functions (time or frequency) are required to characterize ‘two-port’ linear systems. There are many equivalent formulations for these four function parameters. These include the Z-parameters, G-parameters, H-parameters, Y-parameters, S-parameters, etc. The S-parameters are the most commonly used, especially at higher frequencies. They are the only ones that can be measured at high frequencies. A network analyzer measures one or more of these four S-parameters as functions of frequency. (Typically, two (transmission/reflection) at a time or all four at a time.)



Network analyzers:
- measure components, devices, circuits, sub-assemblies
- contain source and receiver
- display ratioed amplitude and phase (frequency or power sweeps)
- offer advanced error correction

Spectrum analyzers:
- measure signal amplitude characteristics (carrier level, sidebands, harmonics, etc.)
- can demodulate (& measure) complex signals
- are receivers only (single channel)
- can be used for scalar component test (no phase) with tracking gen. or ext. source(s)

Fig. 2: Differences between Network and Spectrum Analyzers. From [2].

Roughly speaking, in addition to the forward gain, you also need to know the input resistance, the output resistance, and the reverse gain to characterize a two-port. And all of these can be functions of frequency. Vector network analyzers determine both the amplitude and the phase associated with these functions of frequency. Scalar network analyzers determine only the amplitude.

REFERENCES

[1] R. E. Ziemer and W. H. Tranter, *Principles of Communications*, 5th ed. Hoboken, NJ: John Wiley, 2002.
[2] Agilent Technologies, “Understanding the Fundamental Principles of Vector Network Analysis,” Agilent Technologies, AN 1287-1, 2000. http://cp.literature.agilent.com.
[3] ——, “Exploring the Architectures of Network Analyzers,” Agilent Technologies, AN 1287-2, 2000. http://cp.literature.agilent.com