Bonus 6

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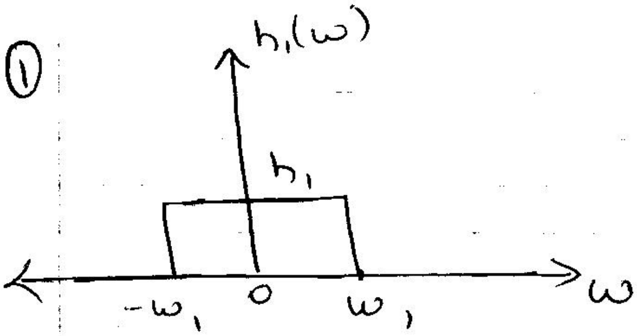
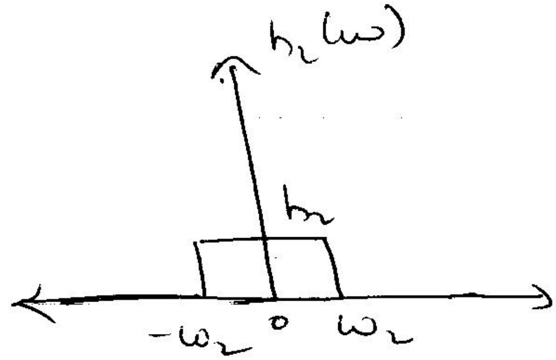
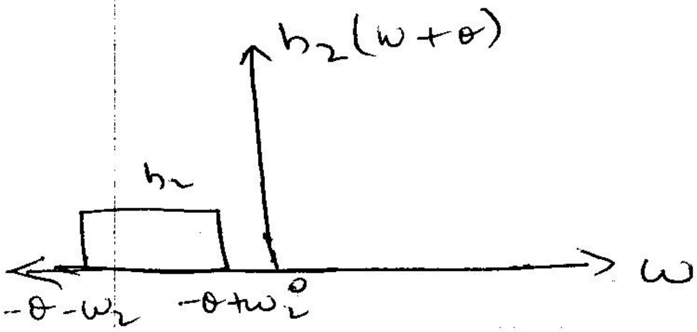
(1)

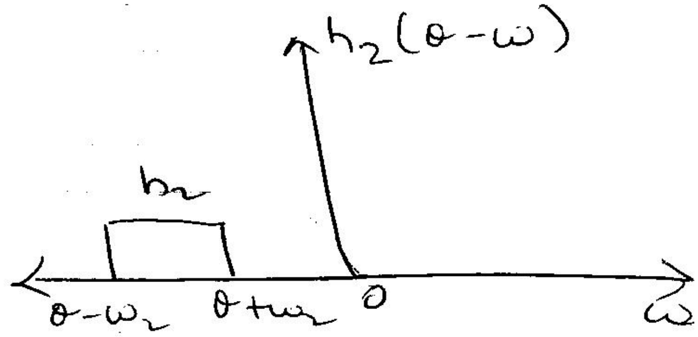


$*$



Lets assume $\omega_1 > \omega_2$, If $\omega_2 > \omega_1$, just invert the convolution.





(I) $\theta + \omega_2 < -\omega_1 \Rightarrow \theta < -\omega_1 - \omega_2$

$y(\theta) = 0.$

(II) $\theta + \omega_2 > -\omega_1 \Rightarrow \theta > -\omega_1 - \omega_2$

$\quad \& \ \theta < \omega_2 - \omega_1.$

$$y(\theta) = \int_{-\omega_1}^{\theta+\omega_2} h_1 h_2 \, d\tau = h_1 h_2 [\theta + \omega_1 + \omega_2]$$

(III) $t > \omega_2 - \omega_1$ & $t < \omega_1 - \omega_2$.

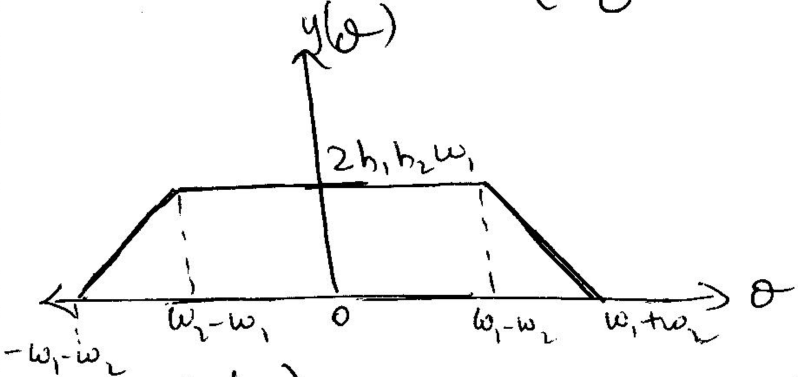
$$y(\theta) = \int_{\theta-\omega_1}^{\theta+\omega_1} h_1 h_2 \, d\tau = h_1 h_2 (2\omega_1)$$

(IV) $t > \omega_1 - \omega_2$ & $t < \omega_1 + \omega_2$.

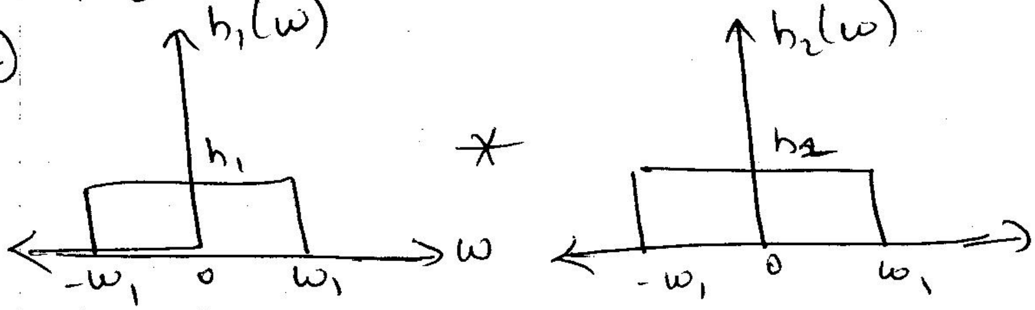
$$y(\theta) = \int_{\theta-\omega_2}^{\omega_1} h_1 h_2 \, d\tau = h_1 h_2 [\omega_1 + \omega_2 - \theta]$$

(V) $t > \omega_1 + \omega_2$, $y(\theta) = 0$.

$$\therefore \quad y(\theta) = \begin{cases} 0, & \theta < -\omega_1 - \omega_2 \\ h_1 h_2 [\theta + \omega_1 + \omega_2], & -\omega_1 - \omega_2 < \theta < \omega_2 - \omega_1 \\ h_1 h_2 \cdot 2\omega_1, & \omega_2 - \omega_1 < \theta < \omega_1 - \omega_2 \\ h_1 h_2 [\omega_1 + \omega_2 - \theta], & \omega_1 - \omega_2 < \theta < \omega_1 + \omega_2 \\ 0, & \theta > \omega_1 + \omega_2 \end{cases}$$



(2)



(I) $\theta < -2\omega_1$ $y(\theta) = 0$.

(II) $\theta > -2\omega_1$ & $\theta < 0$

$$y(\theta) = \int_{-\omega_1}^{\theta+\omega_1} h_1 h_1 \, d\tau = h_1^2 [\theta + 2\omega_1]$$

(III) $\theta > 0$ & $\theta < 2\omega_1$

$$y(\theta) = \int_{\theta-\omega_1}^{\omega_1} h_1^2 \, d\tau = h_1^2 [2\omega_1 - \theta]$$

IV $\theta > 2\omega_1 \quad y(\theta) = 0$

$$\therefore \quad y(\theta) = \begin{cases} 0 & , \theta < -2\omega_1 \\ h_1^2[\theta + 2\omega_1] & , -2\omega_1 < \theta < 0 \\ h_1^2[2\omega_1 - \theta] & , 0 < \theta < 2\omega_1 \\ 0 & , \theta > 2\omega_1 \end{cases}$$