(Review)

$$x(t) = e^{j\omega_0 t} \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ h(t) \leftrightarrow H(\omega)\end{matrix}} \longrightarrow y(t) = H(\omega_0) e^{j\omega_0 t}$$

$$e^{j\omega_0 t} * h(t) = H(\omega_0) e^{j\omega_0 t}$$

- In freq. domain → same result

$H(\omega)$

$$y(t) = e^{j\omega_0 t} * h(t) \xleftrightarrow{\mathcal{F}} Y(\omega) = 2\pi\,\delta(\omega - \omega_0)\ \cancel{H(\omega_0)}$$

$$= H(\omega_0) e^{j\omega_0 t} \xleftrightarrow[\mathcal{F}]{} \quad = H(\omega_0)\, 2\pi\, \delta(\omega - \omega_0)$$

- Using Euler's formula,

$$\cos(\omega_0 t) \longrightarrow \boxed{\begin{matrix}\text{LTI} \\ h(t) \leftrightarrow H(\omega)\end{matrix}} \longrightarrow \begin{matrix} H(\omega_0)\cos(\omega_0 t) \\ = |H(\omega_0)| \cos(\omega_0 t + \angle H(\omega_0)) \end{matrix}$$

$$\sin(\omega_0 t) \longrightarrow \boxed{\quad '' \quad} \longrightarrow |H(\omega_0)| \sin(\omega_0 t + \angle H(\omega_0))$$

$$\left(H(\omega_0) = |H(\omega_0)|\, e^{j\angle H(\omega_0)} \right)$$

polar form

· Further, invoking linearity

$$\sum_{k=-\infty}^{\infty} a_k e^{j\omega_k t} \rightarrow \boxed{h(t)} \rightarrow \sum_{k=-\infty}^{\infty} a_k H(\omega_k) e^{j\omega_k t}$$

(sum of sinewaves)

· Further,

$$\sum_{k=-\infty}^{\infty} A_k \cos(\omega_k t + \theta_k) \rightarrow \boxed{h(t)} \rightarrow \sum_{k=-\infty}^{\infty} A_k |H(\omega_k)| \cos(\omega_k t + \theta_k + \angle H(\omega_k))$$

Def. of FT came from passing sinewave thru LTI system $\Rightarrow$ lead to FT of Impulse Response

$h(t) \overset{\mathcal{F}}{\longleftrightarrow} H(\omega)$: characterizes response of LTI system as a function of freq.

Then, used same def. for signal, but interpret differently.

$$= \mathcal{F}\{x(t)\}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$x(t)$ time-domain

$$x(t) \xleftrightarrow{\mathcal{F}} X(\omega)$$

$X(\omega)$ freq-domain

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \mathcal{F}^{-1}\{X(\omega)\}$$

- $X(\omega)$, or more specifically $|X(\omega)|^2$, characterizes the energy distribution of $x(t)$ as a func. of freq.

(Recall) Parseval's theorem

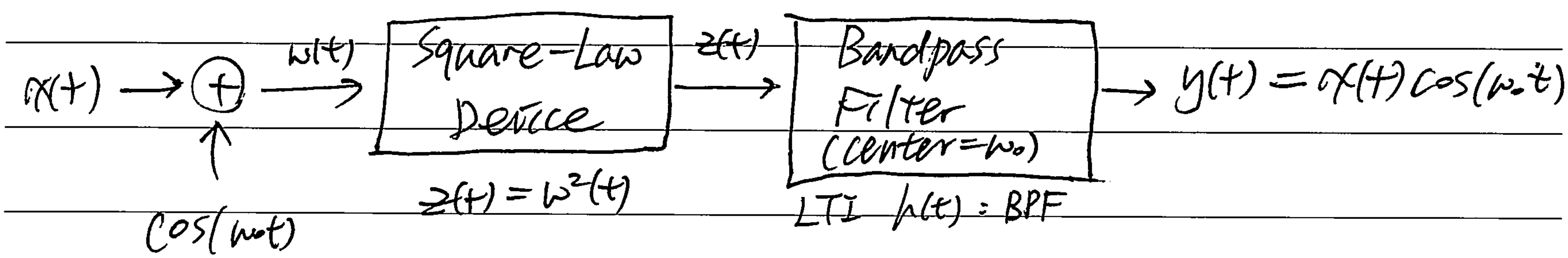
energy $\rightarrow E_x = \int_{-\infty}^{\infty} x^2(t)\, dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2\, d\omega$

- How do we form the product of a signal with a sinewave?

($y(t) = x(t)\cos(\omega_0 t)$?)

→ done to make a signal radiate from an antenna AND to put signals in different freq. bands so they don't interfere.

- trick: put the signal voltage waveform in series with the sinusoidal source, then apply sum of voltages to square-law device. (: involves diode in combination with voltage-follower). Then filter to isolate the product $x(t)\cos(\omega_0 t)$ while filtering the signal components near DC (baseband) and at $2\omega_0$ (twice the freq.)



$$z(t) = \left(x(t) + \cos(\omega_0 t)\right)^2 \qquad \overline{= \frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t)}$$

$$= x^2(t) + 2x(t)\cos(\omega_0 t) + \cos^2(\omega_0 t)$$

$$= \underbrace{\frac{1}{2} + x^2(t)} + \underbrace{2\boxed{x(t)\cos(\omega_0 t)}} + \underbrace{\frac{1}{2}\cos(2\omega_0 t)}$$

spectrally located in freq domain around $\omega = 0$ (baseband)
→ filtered out, not passed by BPF

located near $\underline{\omega = \omega_0}$ in freq domain
→ passed by BPF

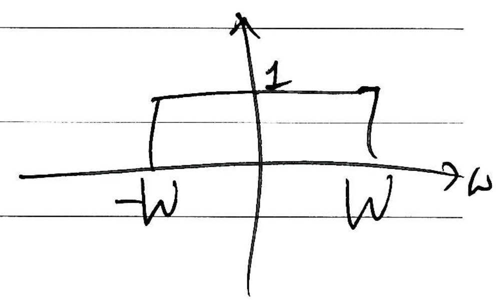
located at $\omega = 2\omega_0$
→ not passed by BPF or rejected.

$$Z(\omega) = \frac{1}{2}\cdot 2\pi\delta(\omega) + \frac{1}{2\pi}X(\omega) * X(\omega) + X(\omega-\omega_0) + X(\omega+\omega_0)$$

$$+ \frac{1}{2}\pi\delta(\omega - 2\omega_0) + \frac{1}{2}\pi\delta(\omega + 2\omega_0)$$

For simplicity, assume $x(t) = \dfrac{\sin(Wt)}{\pi t}$

$$X(\omega) = \mathrm{rect}\left(\frac{\omega}{2W}\right)$$

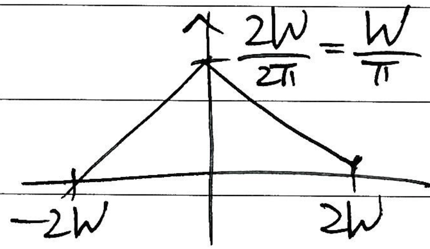


* BPF : Band-Pass Filter

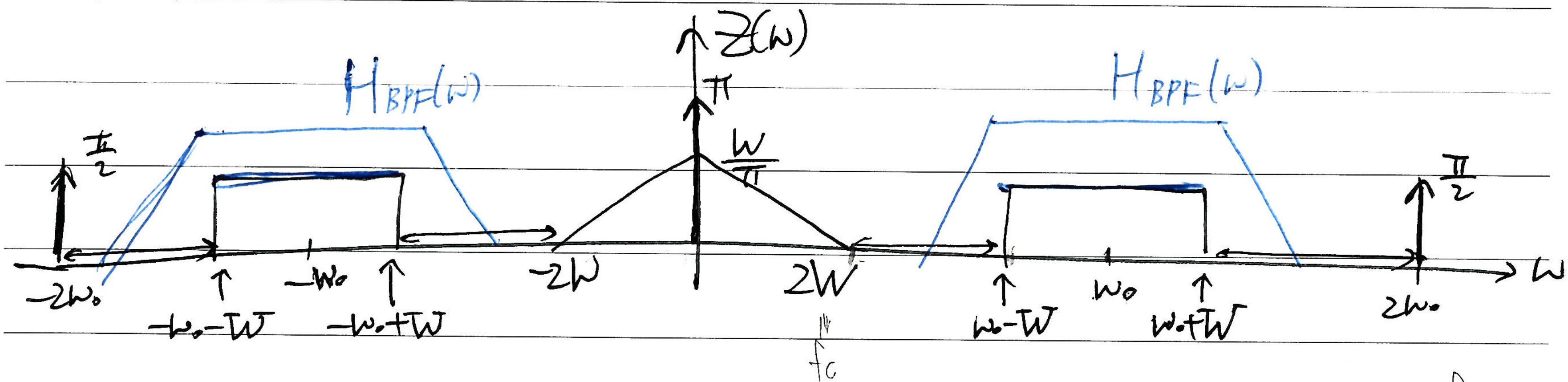
From multiplication property, we have:

$$x^2(t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega) * X(\omega)$$

$$\left(\frac{\sin(Wt)}{\pi t}\right)^2 \overset{\mathcal{F}}{\longleftrightarrow}$$



Assume $\omega_0 \gg W$



$\omega_0 > 2f_c$

- Have flexibility on BPF design:
  - desire magnitude to be flat over $\omega_0 - W < |\omega| < \omega_0 + W$
  - can roll-off to zero over $\begin{cases} 2W < |\omega| < \omega_0 - W \\ \omega_0 + W < |\omega| < 2\omega_0 \end{cases}$

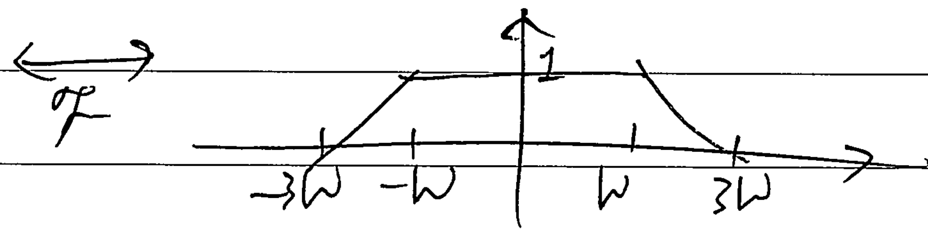
trapezoid in freq. domain

For example:

$$h_{BPF}(t) = \frac{2\pi}{W} \frac{\sin(Wt)}{\pi t} \frac{\sin(2Wt)}{\pi t} \cos(w_0 t)$$

$$= 2\, h_{LPF}(t) \cos(w_0 t)$$

where $h_{LPF}(t) = \frac{\pi}{W} \frac{\sin(Wt)}{\pi t} \frac{\sin(2Wt)}{\pi t}$



Then $h_{BPF}(t) = 2\, h_{LPF}(t) \cos(w_0 t)$

$$\to H_{BPF}(w) = H_{LPF}(w - w_0) + H_{LPF}(w + w_0)$$

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