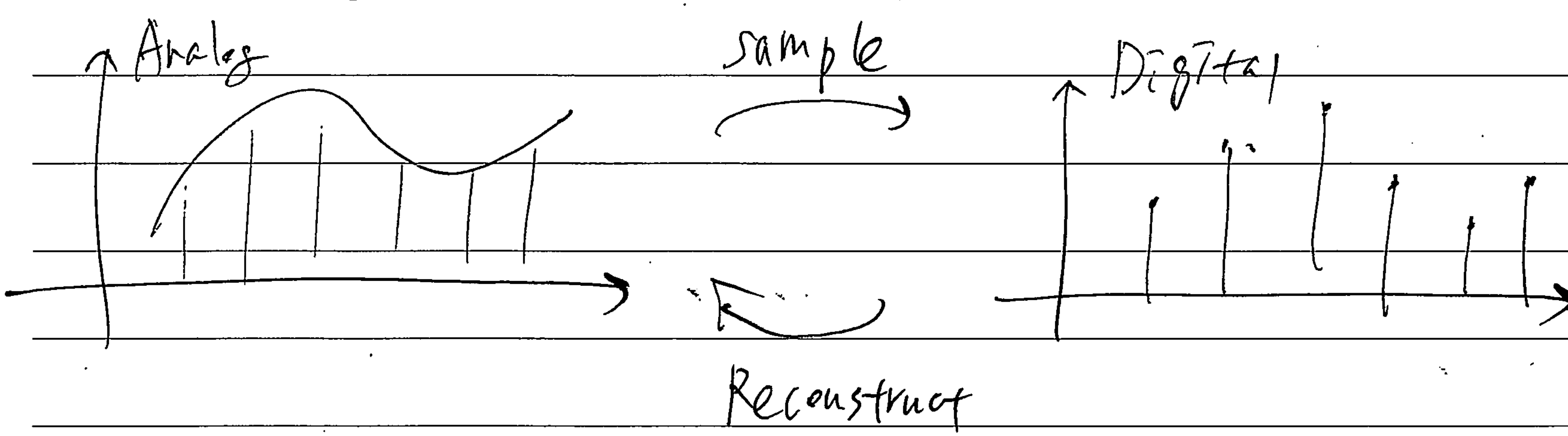
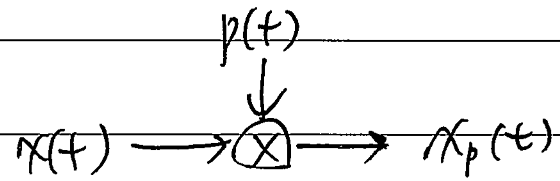
Chap 7. Sampling Theory

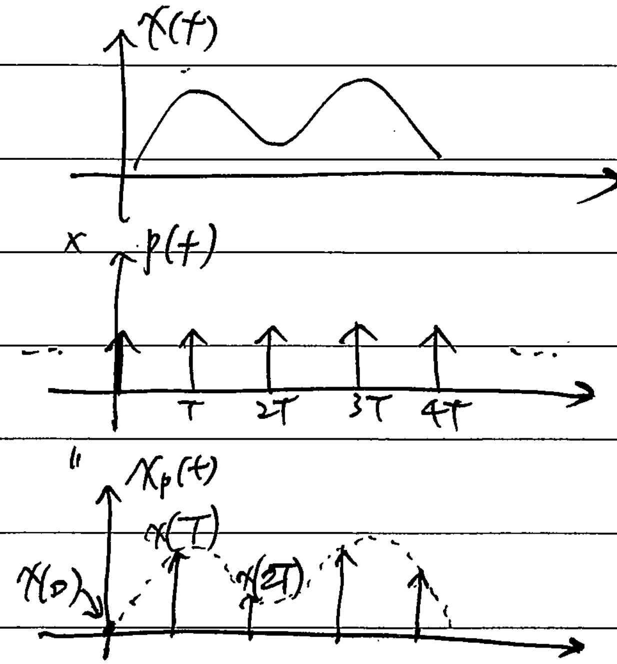
- Basis for the digital revolution.
- Theoretical foundation for practical A/D or D/A converters
- * Initially address the fundamental question of how fast do we have to sample a signal in order to retain FULL information about the signal.
- * "How do we reconstruct the entire analog signal only knowing its values at equi-spaced instants in time?"



- If we multiply $x(t)$ by a periodic train of Dirac Delta Func., we pick off the values at equi-spaced instants in time

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

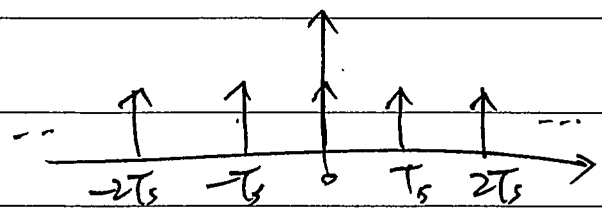


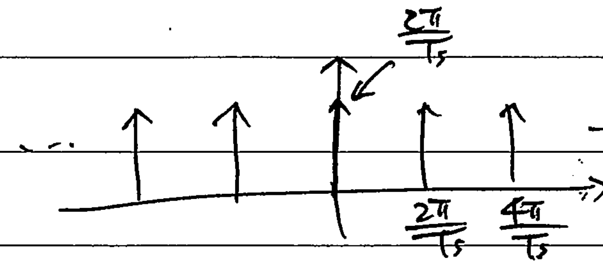


$$x_p(t) = x(t)\, p(t) = x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) = x(nT_s) \sum_{n} \delta(t - nT_s)$$

- To analyze in the freq domain, recall

  - $p(t) = \sum_n \delta(t - nT_s) \xleftrightarrow{\mathcal{F}} P(\omega) = \frac{2\pi}{T_s} \sum_k \delta\left(\omega - k\frac{2\pi}{T_s}\right)$





  - multiplication property of FT

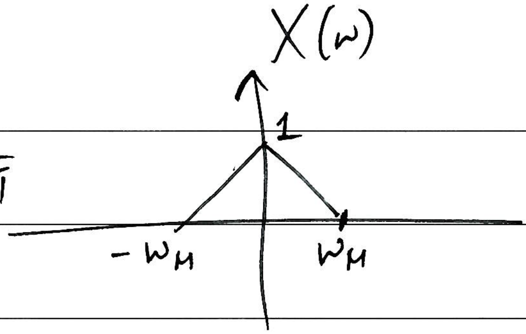
$$x_p(t) = x(t)\, p(t) \xleftrightarrow{\mathcal{F}} X_p(\omega) = \frac{1}{2\pi} X(\omega) * P(\omega)$$

- $X_p(\omega) = \frac{1}{\cancel{2\pi}} X(\omega) * \frac{\cancel{2\pi}}{T_s} \sum_k \delta\left(\omega - k\frac{2\pi}{T_s}\right)$

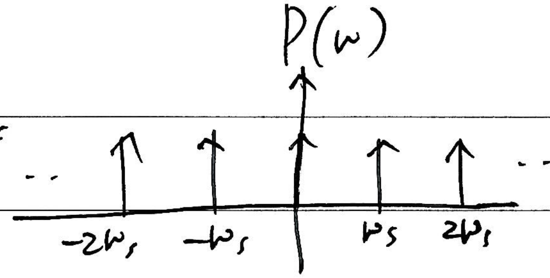
$$= \frac{1}{T_s} \sum_k X(\omega) * \delta\left(\omega - k\frac{2\pi}{T_s}\right)$$

$$= \frac{1}{T_s} \sum_k X\left(\omega - k\frac{2\pi}{T_s}\right)$$

$\frac{1}{2\pi}$



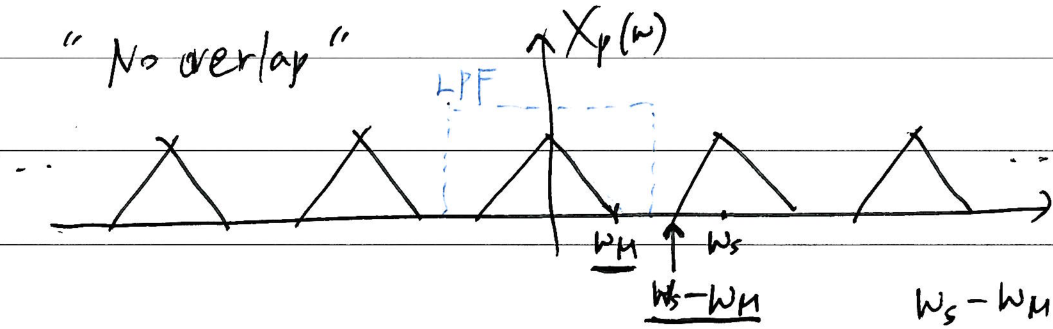
$*$



where $w_s = \frac{2\pi}{T_s}$, T_s : sampling period, $\frac{1}{T_s}$: sampling rate (samples/sec)

w_M : max freq in $x(t)$

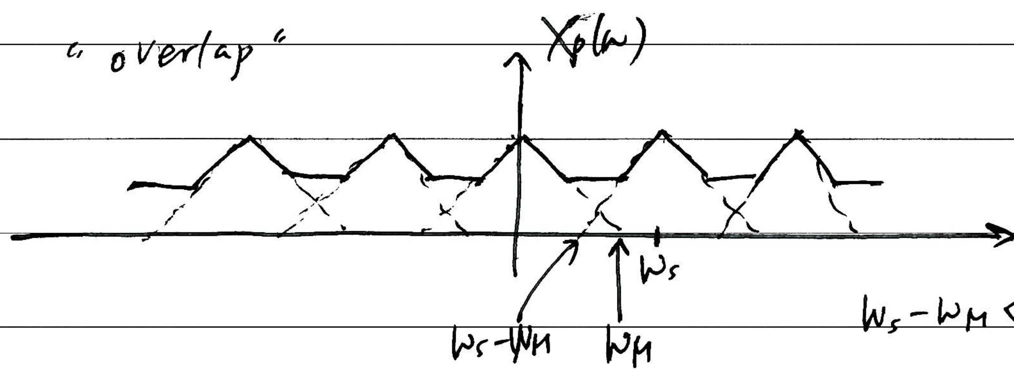
= "No overlap"



for $w_s > 2w_M$ (Nyquist)

$w_s - w_M > w_M$

"overlap"



for $w_s < 2w_M$

$w_s - w_M < w_M$

- If $\omega_s - \omega_M > \omega_M$ (or $\omega_s > 2\omega_M$)

  there is no overlap amongst the spectral replications

  $\Rightarrow$ "no aliasing"

  $\Rightarrow$ $2\omega_M$: Nyquist Rate

- Then, we can recover ~~ogt~~ original signal by lowpass filtering to pass only the original spectrum centered at $\omega=0$ and reject all the other replicas.

- In practice, sample greater than Nyquist rate so that there is a gap between the replica centered at $\omega_s = \frac{2\pi}{T_s}$ and the original spectrum centered at $\omega=0$, so practical lowpass can roll-off from 1 to 0.

(subnote) when $\omega_s = 2\omega_M$