ECE 301
Division 3, Fall 2007
Instructor: Mimi Boutin
Midterm Examination 3

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to answer the 4 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This booklet contains 9 pages. The last four pages contain a table of formulas and properties. You may tear out these pages **once the exam begins**. Each transform and each property is labeled with a number. To save time, you may use these numbers to specify which transform/property you are using when justifying your answer. In general, if you use a fact which is *not* contained in this table, you must explain why it is true in order to get full credit. The only exceptions are the properties of the ROC, which can be used without justification.
4. This is a closed book exam. The only personal items allowed are pens/pencils, erasers and something to drink. Anything else is strictly forbidden.

Itemized Scores
Problem 1:
Problem 2a):
Problem 2b):
Problem 2c):
Problem 3:
Problem 4:

Name:______________________________

Email:______________________________

Signature:____________________________

(10 pts) **1.** State the sampling theorem. (You may use your own words but be precise!)

2. Consider the signal $x(t) = \cos(3\pi t) + \cos(5\pi t)$.
(5 pts) a) Is $x(t)$ band-limited? If so, what is the Nyquist rate for this signal (Justify your answers.)

(10 pts) b) Suppose the signal $x(t)$ is modulated by the carrier signal $c(t) = e^{10j\pi t}$. Sketch the graph of the modulated signal in the frequency domain. Can one recover $x(t)$ from the modulated signal? If you answered yes, draw the diagram of a system which could be used to demodulate the signal. If you answered no, explain why.

(10 pts) c) Suppose the signal $x(t)$ is modulated by the carrier signal $c(t) = \cos(10\pi t)$. Sketch the graph of the modulated signal in the frequency domain. Can one recover $x(t)$ from the modulated signal? If you answered yes, draw the diagram of a system which could be used to demodulate the signal. If you answered no, explain why.

(20 pts) **3. DT processing of CT signals.** A CT signal $x(t)$ with Nyquist rate equal to 2000π is the input of a CT system consisting of a low-pass filter with gain 3 and cut-off frequency 500π. Draw the diagram of an *equivalent* system which would sample $x(t)$, process the samples using a DT system, and reconstruct a CT signal from the processed samples. (Don't forget to specify an appropriate frequency for the sampling as well as the frequency response of the DT system.) Illustrate all the steps of the system by sketching (in the frequency domain) the graph of an input signal together with the graphs of the signals obtained at each of the intermediate steps of the system.

(15 pts) **4.** Compute the Laplace transform of the following signal *without using the table of Laplace transform pairs*:

$$x(t) = e^{-2t}u(t) + e^{5t}u(-t).$$

Table

Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{1}$$

$$a_k = \frac{1}{T}\int_0^T x(t)e^{-jk\left(\frac{2\pi}{T}\right)t}dt \tag{2}$$

Some CT Fourier series

Signal	a_k	
$e^{j\omega_0 t}$	$a_1 = 1, a_k = 0$ else.	(3)
Periodic square wave $x(t) = \begin{cases} 1, & \lvert t\rvert < T_1 \\ 0 & T_1 < \lvert t\rvert < \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\dfrac{\sin k\omega_0 T_1}{k\pi}$	(4)
$\sum_{n=-\infty}^{\infty} \delta(t-nT)$	$a_k = \dfrac{1}{T}$	(5)

Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \tag{6}$$

$$a_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-jk\left(\frac{2\pi}{N}\right)n} \tag{7}$$

CT Fourier Transform

$$\begin{aligned} \text{F.T. : } \mathcal{X}(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt && (8) \\ \text{Inverse F.T.: } x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}d\omega && (9) \end{aligned}$$

Properties of CT Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

	Signal	F.T.	
Linearity:	$ax(t) + by(t)$	$a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$	(10)
Time Shifting:	$x(t - t_0)$	$e^{-j\omega t_0}\mathcal{X}(\omega)$	(11)
Frequency Shifting:	$e^{j\omega_0 t}x(t)$	$\mathcal{X}(\omega - \omega_0)$	(12)
Time and Frequency Scaling:	$x(at)$	$\frac{1}{\lvert a\rvert}\mathcal{X}\left(\frac{\omega}{a}\right)$	(13)
Multiplication:	$x(t)y(t)$	$\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$	(14)
Convolution:	$x(t) * y(t)$	$\mathcal{X}(\omega)\mathcal{Y}(\omega)$	(15)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$j\omega\mathcal{X}(\omega)$	(16)

Some CT Fourier Transform Pairs

$$\begin{aligned} e^{j\omega_0 t} &\xrightarrow{\mathcal{F}} 2\pi\delta(\omega - \omega_0) && (17) \\ 1 &\xrightarrow{\mathcal{F}} 2\pi\delta(\omega) && (18) \\ \frac{\sin Wt}{\pi t} &\xrightarrow{\mathcal{F}} u(\omega + W) - u(\omega - W) && (19) \\ \delta(t) &\xrightarrow{\mathcal{F}} 1 && (20) \\ u(t + T_1) - u(t - T_1) &\xrightarrow{\mathcal{F}} \frac{2\sin(\omega T_1)}{\omega} && (21) \\ \sum_{n=-\infty}^{\infty}\delta(t - nT) &\xrightarrow{\mathcal{F}} \frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta(\omega - \frac{2\pi k}{T}) && (22) \end{aligned}$$

DT Fourier Transform

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{23}$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \tag{24}$$

Properties of DT Fourier Transform

Let $x(t)$ be a signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

	Signal	F.T.	
Linearity:	$ax[n] + by[n]$	$a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$	(25)
Time Shifting:	$x[n - n_0]$	$e^{-j\omega n_0}\mathcal{X}(\omega)$	(26)
Frequency Shifting:	$e^{j\omega_0 n}x[n]$	$\mathcal{X}(\omega - \omega_0)$	(27)
Time Reversal:	$x[-n]$	$\mathcal{X}(-\omega)$	(28)
Time Exp.:	$x_k[n] = \begin{cases} x[\frac{n}{k}], & \text{if } k \text{ divides } n \\ 0, & \text{else.} \end{cases}$	$\mathcal{X}(\omega)$	(29)
Multiplication:	$x[n]y[n]$	$\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$	(30)
Convolution:	$x[n] * y[n]$	$\mathcal{X}(\omega)\mathcal{Y}(\omega)$	(31)
Differencing in Time:	$x[n] - x[n-1]$	$(1 - e^{-j\omega})\mathcal{X}(\omega)$	(32)

Some DT Fourier Transform Pairs

$$\sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \xrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N}) \tag{33}$$

$$1 \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) \tag{34}$$

$$\tag{35}$$

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \tag{36}$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}} \tag{37}$$

$$\tag{38}$$

Laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt \tag{39}$$

Properties of Laplace Transform

Let $x(t)$, $x_1(t)$ and $x_2(t)$ be three CT signals and denote by $X(s)$, $X_1(s)$ and $X_2(s)$ their respective Laplace transform. Let R be the ROC of $X(s)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(s)$.

	Signal	L.T.	ROC	
Linearity:	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$	(40)
Time Shifting:	$x(t - t_0)$	$e^{-st_0}X(s)$	R	(41)
Shifting in s:	$e^{s_0 t}x(t)$	$X(s - s_0)$	$R + s_0$	(42)
Conjugation:	$x^*(t)$	$X^*(s^*)$	R	(43)
Time Scaling:	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{s}{a}\right)$	aR	(44)
Convolution:	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$	(45)
Differentiation in Time:	$\frac{d}{dt}x(t)$	$sX(s)$	At least R	(46)
Differentiation in s:	$-tx(t)$	$\frac{dX(s)}{ds}$	R	(47)
Integration :	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \mathcal{R}e\{s\} > 0$	(48)

Some Laplace Transform Pairs

Signal	LT	ROC	
$u(t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} > 0$	(49)
$-u(-t)$	$\frac{1}{s}$	$\mathcal{R}e\{s\} < 0$	(50)
$u(t)\cos(\omega_0 t)$	$\frac{s}{s^2 + \omega_0^2}$	$\mathcal{R}e\{s\} > 0$	(51)
$e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\mathcal{R}e\{s\} > -\alpha$	(52)
$-e^{-\alpha t}u(-t)$	$\frac{1}{s + \alpha}$	$\mathcal{R}e\{s\} < -\alpha$	(53)
$\delta(t)$	1	all s	(54)