ECE 438 **Assignment No. 2** **Spring 2009**

1. For each of the following signals, compute the DTFT $X(\omega)$. Simplify your answer as much as possible.

a. $x[n] = 2\sin(\pi(n-1)/3)$,

b. $x[n] = \delta[n+1]-\delta[n-1]$,

c. $x[n] = e^{n}u[-n]$.

2. Find an expression for the DTFT $Y(\omega)$ of the output in terms of the DTFT $X(\omega)$ of the input, when y[n] and x[n] are related by (Simplify your answer as much as possible, and assume that x[n] is real-valued.):

a. $y[n] = (-1)^{n}x[n]$,

b. $y[n] = x[n] - x[-n]$,

c. $y[n] = x[n+1] - x[n-1]$

3. Perform the convolution of the following pairs of signals:

a. $\left(\frac{1}{2}\right)^{-|n|}\left(u[n+4]-u[n-5]\right)$ and $u[n]-u[n-11]$

b. $u[n+10]-u[n]$ and $u[n]-u[n-10]$

4. Consider a DT LTI system described by the following equation

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

Find the response of this system to the input

$$x[n] = \begin{cases} -2, & n=-2, \\ 1, & n=0, \\ -2 & n=2, \\ 0, & \text{else.} \end{cases}$$

by the following approaches:

a. directly substitute $x[n]$ into the difference equation describing the system;

b. find the impulse response $h[n]$ and convolve it with $x[n]$;

c. find the frequency response $H(\omega)$ by the following two approaches:

i. apply the input $e^{j\omega n}$ to the difference equation describing the system,

ii. find the DTFT of the impulse response,

verify that both methods lead to the same result, then find the DTFT $X(\omega)$ of the input, multiply it by $H(\omega)$ to yield the DTFT $Y(\omega)$ of the output, and finally calculate the inverse DTFT $y[n]$.

Hints:

i. There is no need to simplify the frequency response or the DTFT of the input.

ii. To evaluate the inverse DTFT of $Y(\omega)$, simply put it in the series form $Y(\omega) = \sum_n y[n]\, e^{-j\omega n}$, and identify the terms $y[n]$ in the series.

d. Verify that all three approaches for finding $y[n]$ lead to the same result.

5. Consider the DT LTI system described by the difference equation

$$y[n] = \{x[n] + x[n-1] + x[n-2] + \cdots + x[n-(N-1)]\} / N$$

a. Find expressions for the magnitude $|H(\omega)|$ and phase $\arg[H(\omega)]$ of the frequency response for this system.

b. Use Matlab to plot these quantities for $-\pi \le \omega \le \pi$ for the following values of the constant N:

i. $N = 3$,

ii. $N = 9$,

iii. $N = 21$.

c. Discuss the significance of your results. What happens as N increases?