More about problems 32 and 34 on p. 287.

32. To see that the set of vectors $\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix}$ in $\mathbb{R}^3$ such that $\begin{cases} 3u_1 - 2u_2 + V_3 = 0 \\ 4u_1 + 5u_2 = 0 \end{cases}$

is a subspace of $\mathbb{R}^3$, we only need to check two things (because of the Subspace Test).

1) We need to see that the set is <u>closed</u> under addition. So, suppose

$$\vec{u} = \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} \text{ and } \vec{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} \text{ satisfy the}$$

system, i.e., $\begin{cases} 3u_1 - 2u_2 + u_3 = 0 \\ 4u_1 + 5u_2 = 0 \end{cases}$ and $\begin{cases} 3v_1 - 2v_2 + v_3 = 0 \\ 4v_1 + 5v_2 = 0 \end{cases}$

Add those together to see $\begin{cases} 3(u_1+v_1) - 2(u_2+v_2) + (u_3+v_3) = 0 \\ 4(u_1+v_2) + 5(u_2+v_2) = 0 \end{cases}$

This shows that $\vec{u} + \vec{v} = \begin{pmatrix} u_1+v_1 \\ u_2+v_2 \\ u_3+v_3 \end{pmatrix}$ satisfies the system.

2) Next, we need to see that the set is closed under scalar multiplication. Suppose $\vec{u}$ satisfies the system. Multiply both rows of the system by c to see that $c\vec{u}$ also satisfies the system. We are done.

34. Anything you do to see that the set is not closed under addition or scalar multiplication will suffice to show that the set is <u>not</u> a vector subspace of $\mathbb{R}^n$.

<u>EX</u>:

$$\underset{\text{in}}{\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}} + \underset{\text{in}}{\begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}} = \underset{\text{out}}{\begin{pmatrix} 2 \\ 2 \\ \vdots \\ 2 \end{pmatrix}}$$

Not closed under +

<u>or</u>

$$\underset{\text{zero}}{0} \cdot \underset{\text{in}}{\begin{pmatrix} 1 \\ -1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}} = \underset{\text{out}}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}}$$

Not closed under scalar mult.

Also, no zero vector in set.