# E302 HOMEWORK-2
## SOLUTIONS

(a) $P(X=0) = P(T_0 \cap R_0) + P(T_2 \cap R_0)$

$$= \frac{1}{3}\cdot(0.95) + \frac{1}{3}\cdot(0.93)$$

$$= \frac{47}{75}$$

Where $T_X$ : Transmitting $X$

$R_X$ : Receiving $X$.

$P(X=1) = P(T_0 \cap R_1) + P(T_1 \cap R_1)$

$$= \frac{1}{3}\cdot(0.05) + \frac{1}{3}\cdot(0.8)$$

$$= \frac{17}{60}$$

$P(X=2) = P(T_1 \cap R_2) + P(T_2 \cap R_2)$

$$= \frac{1}{3}\cdot(0.2) + \frac{1}{3}(0.07)$$

$$= \frac{9}{100}$$

) $P(\text{error}) = P(T_0 \cap R_1) + P(T_1 \cap R_2) + P(T_2 \cap R_0)$

$$= \frac{1}{3}(0.05) + \frac{1}{3}(0.2) + \frac{1}{3}(0.93)$$

$$= \frac{59}{150}$$

$$P(T_1/\text{error}) = \frac{P(\text{error}/T_1)\cdot P(T_1)}{P(\text{error})}$$

$$= \frac{P(R_2)\cdot P(T_1)}{P(\text{error})}$$

$$= \frac{0.2 \times \frac{1}{3}}{\frac{59}{150}}$$

$$= \frac{10}{59}$$

(2) A : Urn 1 is selected

B : A black ball is observed

$P(A) = 0.5$

$P(B) = P($Ur1 is selected & a black ball is observed$) + P($Ur2 is selected & a black ball is observed$)$

assume the probability of finding a black ball in Urn 1 $= p$

and Urn 2 $= q$

$$P(B) = \frac{1}{2}p + \frac{1}{2}q = \frac{p+q}{2}$$

For A & B to be independent

$$P(A/B) = P(A)$$

$$\therefore \; P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)} = \frac{p \cdot \frac{1}{2}}{\frac{p+q}{2}} = \frac{1}{2}$$

$$\Rightarrow p = q$$

(3) $P($chip being defective$) = 0.05$

$X = \#$ of chips not defective ; note : X is a binomial R.V

if $n=8$ $\quad P(X=8) = {}^8C_8 \cdot (0.95)^8 = 0.663 < 0.9$

if $n=9$ $\quad P(X=8) + P(X=9) = {}^9C_8 \cdot (0.05)(0.95)^8 + {}^9C_9 \cdot (0.95)^9 = 0.9287 > 0.9$

$\therefore$ He needs to buy 9 chips

(4) X : # of cards she draws : note : X is a geometric R.V

$P(X=n)$ where $1 \leq n \leq 53$ is

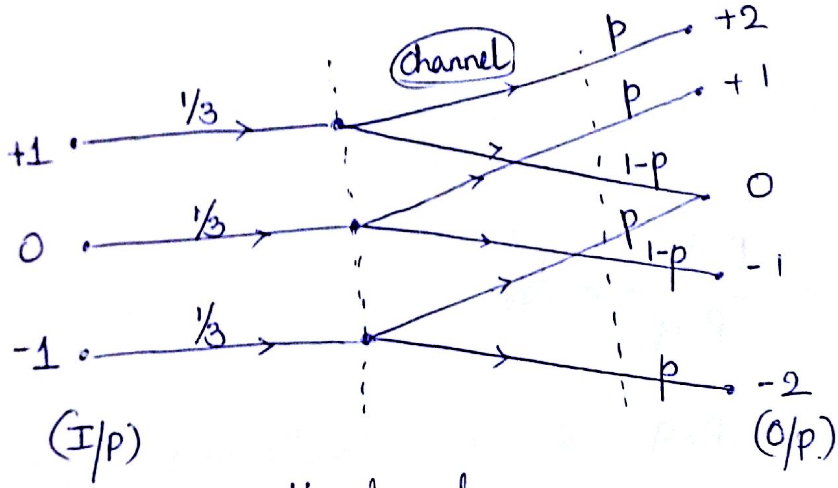
$= (\text{She draws no joker})_1 \times (\text{She draws no joker})_2 \times \cdots \times (\text{She draws a joker})_n$

$= \frac{\cancel{52}}{53} \times \frac{\cancel{51}}{\cancel{52}} \times \cdots \times \frac{\cancel{53-n+1}}{53-n+2} \times \frac{1}{\cancel{53-n+1}}$

$= \frac{1}{53}$

$\therefore$ PMF $P(X) = \begin{cases} \frac{1}{53}, & 1 \leq X \leq 53 \\ 0, & \text{elsewhere} \end{cases}$

(5)




(a) X : output of the channel

$P(X=2) = P/3$

$P(X=1) = P/3$

$P(X=0) = \frac{1}{3}(1-P) + \frac{1}{3}(P) = \frac{1}{3}$

$P(X=-1) = \frac{1}{3}(1-P)$

$P(X=-2) = \frac{1}{3}(1-P)$

$\therefore$ PMF $P(X) = \begin{cases} P/3, & X = 1, 2 \\ 1/3, & X = 0 \\ \frac{1}{3}(1-P), & X = -1, -2 \\ 0, & \text{elsewhere} \end{cases}$

(b) If we assume $0 < P < 1$, then $P(X=0)$ is the most likely value as $P/3$ & $\frac{1}{3}(1-P)$ are both less than $\frac{1}{3}$ and hence doesn't depend on the value of P.

But if we assume $0 \leq P \leq 1$, then the most likely value depends on P as, now we can't say which value of X has the highest probability.

(6) $f_X(x) = \begin{cases} c(1-e^{0.2(x-3)}) , & -2 \leq x \leq 3 \\ 0 , & \text{elsewhere} \end{cases}$

(a) we know that $\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

$\therefore \int_{-2}^{3} c\cdot(1-e^{0.2(x-3)})\,dx = 1$

$C = 0.54365$

(b) $P(-1 < X \leq 2) = \int_{-1}^{2} f_X(x)\,dx$

$= \int_{-1}^{2} (0.54365)\cdot(1-e^{0.2(x-3)})\,dx$

$= 0.6268$

(c) $P((X-1)^2 < 4) = P(-1 < X < 3)$

$= \int_{-1}^{3} f_X(x)\,dx$

$= 0.6777$

(d) $F_X(x) = \int_{-3}^{x} f_X(x)\,dx$

$= \int_{-3}^{x} (0.54365)(1-e^{0.2(x-3)})\,dx$

$= 0.54365x - 0.54365\cdot\left(\frac{e^{0.2x-0.6}}{0.2}\right)\Big|_{-3}^{x}$

$= (0.54365x + 1.63095) - 2.71825\left[e^{0.2x-0.6} - e^{-1.2}\right]$