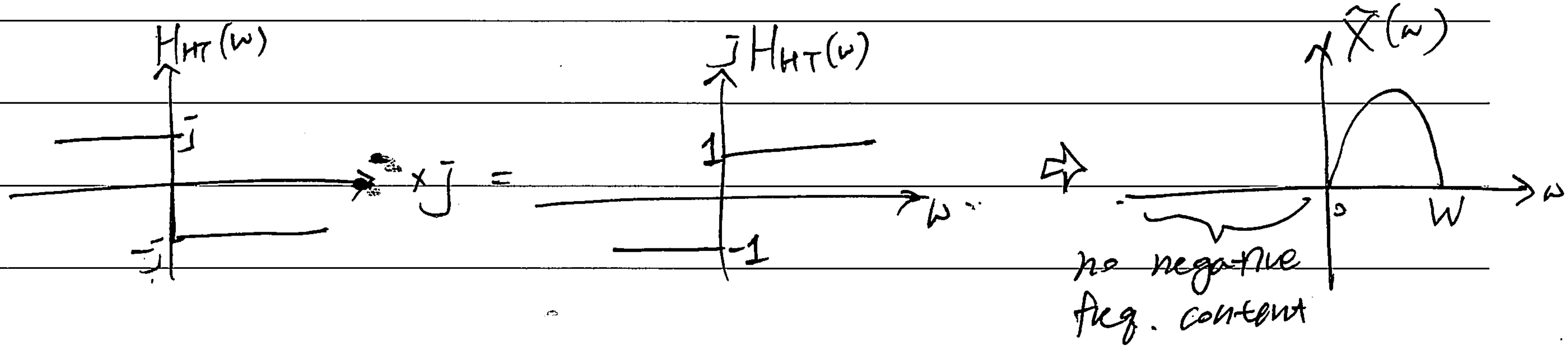
(Review) SSB Modulation. 7/20 ①

$$\tilde{x}(t) = x(t) + j\,\hat{x}(t) \qquad\qquad \tilde{X}(\omega) = X(\omega) + j\,\hat{X}(t)$$

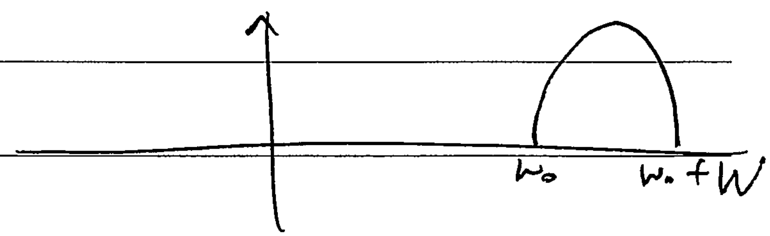
$$= x(t) + j\,x(t) * h_{HT}(t) \quad \xleftrightarrow{\mathcal{F}} \quad = X(\omega) + j\,X(\omega)\,H_{HT}(\omega)$$

$$= x(t) * \left(\delta(t) + j\,h_{HT}(t)\right) \qquad\qquad = X(\omega)\left(\underline{1 + j\,H_{HT}(\omega)}\right)$$



- Note, though, that $\tilde{x}(t)$ is complex-valued and we can only transmit real-valued signals.

$$v(t) = \tilde{x}(t)\,e^{j\omega_0 t} \xleftrightarrow{\mathcal{F}} V(\omega) = \tilde{X}(\omega - \omega_0)$$



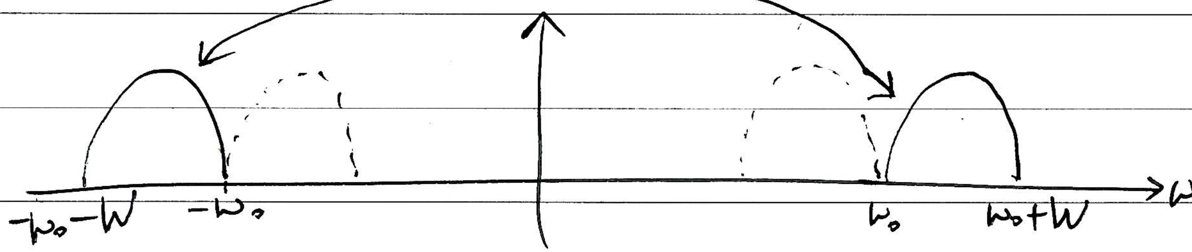
* SSB: Single Sideband

• Consider transmitting real part of of $v(t)$

$$y(t) = Re\{v(t)\} = \frac{1}{2}v(t) + \frac{1}{2}v^*(t)$$

"$x^*(t) \overset{\mathcal{F}}{\longleftrightarrow} X^*(-\omega)$"

$$\overset{\mathcal{F}}{\longleftrightarrow} Y(\omega) = \frac{1}{2}V(\omega) + \underbrace{\frac{1}{2}V^*(-\omega)}_{\text{creates negative freq. content.}}$$



• What is real part of $v(t)$?

$$y(t) = Re\{v(t)\} = Re\{\tilde{x}(t)e^{j\omega_0 t}\}$$

$$= Re\{(x(t) + j\hat{x}(t))(\cos(\omega_0 t) + j\sin(\omega_0 t))\}$$

$$= x(t)\cos(\omega_0 t) - \hat{x}(t)\sin(\omega_0 t)$$

↳ This is the real-valued signal that is transmitted!

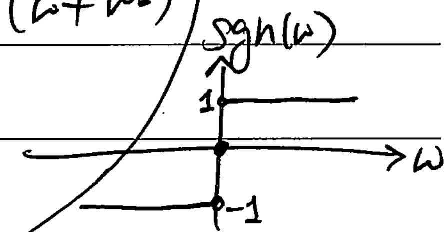
Alternative Derivation

$$y(t) = x(t)\cos(\omega_0 t) - \hat{x}(t)\sin(\omega_0 t) \longleftrightarrow Y(\omega) = ?$$

$$* \quad x(t)\cos(\omega_0 t) \xleftrightarrow{\mathcal{F}} \left(\frac{1}{2}X(\omega-\omega_0) + \frac{1}{2}X(\omega+\omega_0)\right)$$

$$\hat{x}(t)\sin(\omega_0 t) \xleftrightarrow{\mathcal{F}} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0)$$

substitute $\hat{X}(\omega) = -j\,\mathrm{sgn}(\omega)X(\omega)$



$$\hat{x}(t)\sin(\omega_0 t) \xleftrightarrow{\mathcal{F}} \left(-\frac{1}{2}\mathrm{sgn}(\omega-\omega_0)X(\omega-\omega_0) + \frac{1}{2}\mathrm{sgn}(\omega+\omega_0)X(\omega+\omega_0)\right)$$

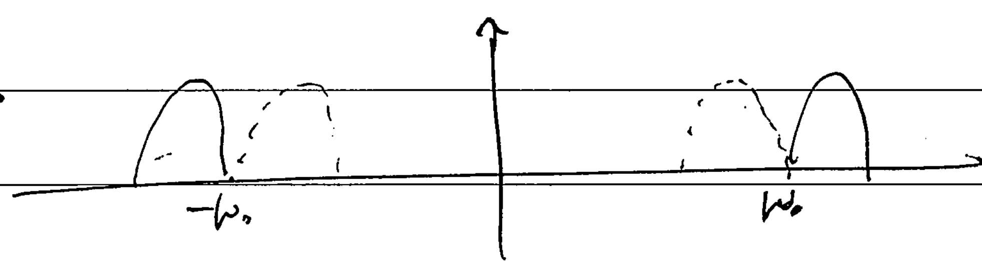
$\oplus$

Thus,
$$Y(\omega) = \frac{1}{2}\underline{\left(1+\mathrm{sgn}(\omega-\omega_0)\right)}X(\omega-\omega_0) + \underline{\frac{1}{2}\left(1-\mathrm{sgn}(\omega+\omega_0)\right)}X(\omega+\omega_0)$$

$$\frac{1}{2}\left(1+\mathrm{sgn}(\omega-\omega_0)\right) = \begin{cases} 1, & \omega > \omega_0 \\ 0, & \omega < \omega_0 \end{cases}$$

$$\frac{1}{2}\left(1+\mathrm{sgn}(\omega+\omega_0)\right) = \begin{cases} 1, & \omega < -\omega_0 \\ 0, & \omega > \omega_0 \end{cases}$$

$\rightarrow$



Practice Problem.

$\nearrow$ odd function $\quad x(-t) = -x(t)$

$$x(t) = \frac{1}{2}\left\{ \frac{\sin\left(10\left(t-\frac{\pi}{10}\right)\right)}{\pi\left(t-\frac{\pi}{10}\right)} - \frac{\sin\left(10\left(t+\frac{\pi}{10}\right)\right)}{\pi\left(t+\frac{\pi}{10}\right)} \right\}$$

$$\xleftrightarrow{\mathcal{F}} X(\omega) = \mathrm{rect}\left(\frac{\omega}{20}\right)\left\{ \frac{1}{2} e^{-j\omega\frac{\pi}{10}} - \frac{1}{2} e^{j\omega\frac{\pi}{10}} \right\}$$

Using :

$$\frac{\sin(Wt)}{\pi t} \xleftrightarrow{\mathcal{F}} \mathrm{rect}\left(\frac{\omega}{2W}\right)$$

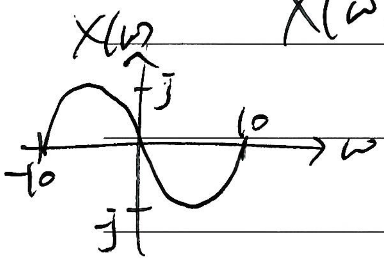
$$x(t-t_0) \xleftrightarrow{\mathcal{F}} X(\omega)\, e^{-j\omega t_0}$$

$e^{jx} = \cos x + j\sin x$

$e^{-jx} = \cos x - j\sin x$

$\frac{e^{jx} - e^{-jx}}{2} = \cancel{2} j \sin x$

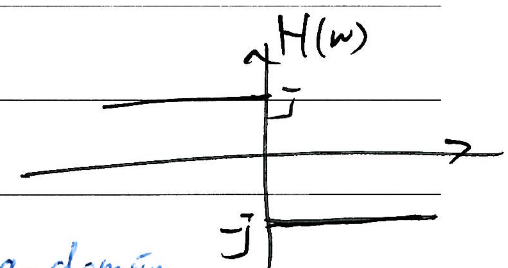
$$X(\omega) = -j \sin\left(\pi \frac{\omega}{10}\right) \mathrm{rect}\left(\frac{\omega}{20}\right)$$



"purely imaginary" since $x(t)$ is odd func.

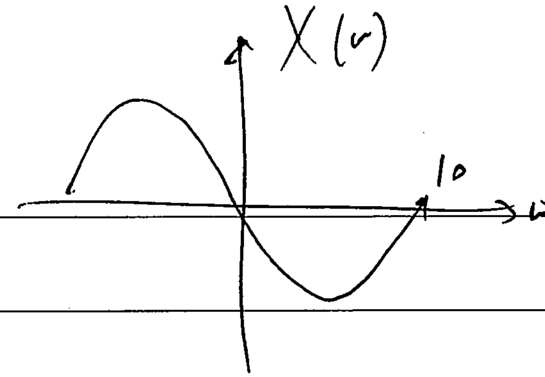
Next, create $\tilde{x}(t) = x(t) + j\hat{x}(t)$ $\quad (\hat{x}(t) = x(t) * h(t))$

$$h(t) = \frac{1}{\pi t} \xleftrightarrow{\mathcal{F}} H(\omega) = \begin{cases} j, & \omega < 0 \\ -j, & \omega > 0 \end{cases}$$



"Hilbert Transformer"

odd func. in time-domain $\longrightarrow$ purely imaginary in freq-domain

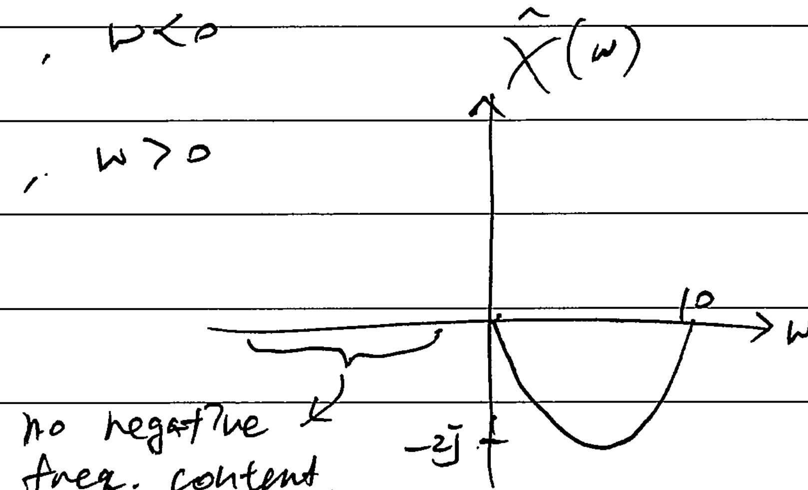


Take $\mathcal{F}T$ of $\tilde{x}(t) = x(t) + j\,\hat{x}(t)$

$$\tilde{X}(\omega) = X(\omega) + j\,\hat{X}(\omega)$$

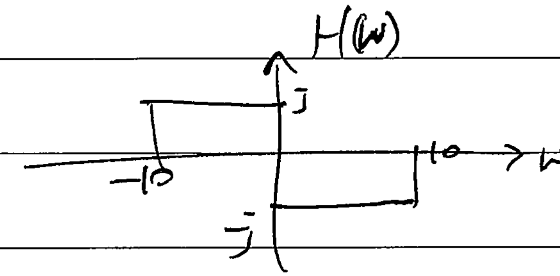
$$= \begin{cases} X(\omega) + j\left(jX(\omega)\right), & \omega < 0 \\ X(\omega) + j\left(-jX(\omega)\right), & \omega > 0 \end{cases}$$

$$= \begin{cases} 0, & \omega < 0 \\ 2X(\omega), & \omega > 0 \end{cases}$$



- Also consider.

$$h(t) = \frac{2\sin(5t)}{\pi t}\sin(5t) \overset{\mathcal{F}}{\longleftrightarrow}$$



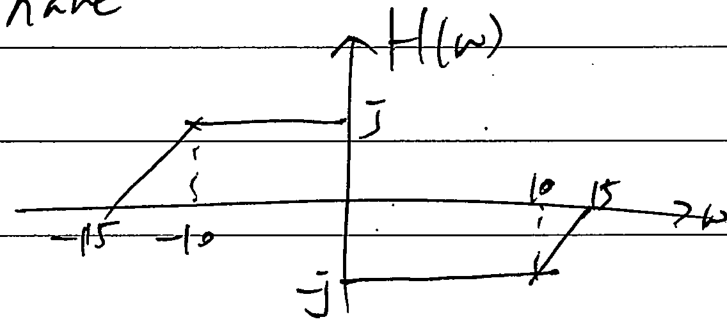
$$\left(x(t)\sin(\omega_0 t) \overset{\mathcal{F}}{\longleftrightarrow} \frac{1}{2j}X(\omega-\omega_0) - \frac{1}{2j}X(\omega+\omega_0) \right)$$

If we use this $h(t)$ to make $\hat{x}(t)$ (or $\hat{X}(\omega)$), would it be different from before for the original signal $x(t)$?

: No. They would be the same.

- How about if we have

$h(t) \overset{\mathcal{F}}{\longleftrightarrow}$



?

$\hat{x}(t) = x(t) + j\hat{x}(t)$ would still be the same. with no negative freq. content.

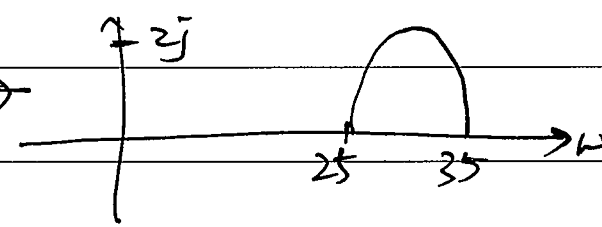
- Next, plot the FT of

$$y(t) = x(t)\cos(25t) - \hat{x}(t)\sin(25t)$$

$$\uparrow \mathrm{Re}\{\hat{x}(t)\, e^{j25t}\} = \mathrm{Re}\{(x(t) + j\hat{x}(t))(\cos(25t) + j\sin(25t))\}$$

(Now recall: $e^{j\omega_0 t}\hat{x}(t) \overset{\mathcal{F}}{\longleftrightarrow} \hat{X}(\omega - \omega_0)$)

Thus, $\hat{x}(t)\, e^{j25t} \overset{\mathcal{F}}{\longleftrightarrow}$

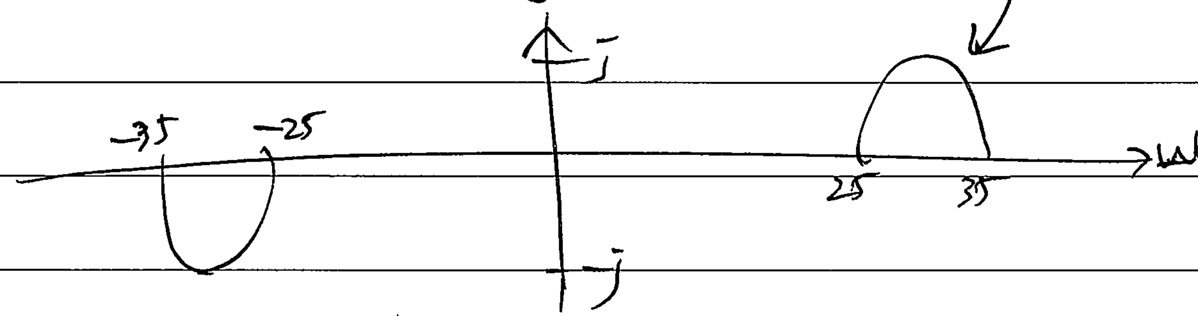


- What does taking the real part of a signal in time do in the freq. domain?

→ since $x^*(t) \xleftrightarrow{\mathcal{F}} X^*(-\omega)$,

$Re\{x(t)\} = \frac{1}{2}x(t) + \frac{1}{2}x^*(t) \xleftrightarrow{\mathcal{F}} \underline{\frac{1}{2}X(\omega)} + \frac{1}{2}X^*(-\omega)$.

Thus, the FT of $y(t) = x(t)\cos(25t) - \hat{x}(t)\sin(25t)$



Expressing the FT and I-FT in terms of Hz

(Hertz = $\frac{\text{cycles}}{\text{sec}}$)

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\underline{e^{j\omega t}\,d\omega}$$

" $\omega = 2\pi f$ " change of variables.

$d\omega = 2\pi\, df$

$$\omega \Big|_{-\infty}^{\infty} \rightarrow f \Big|_{-\infty}^{\infty}$$

$$x(t) = \cancel{\frac{1}{2\pi}}\int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, \cancel{2\pi}\, df$$

Leads to FT and I-FT in terms of Hz

$\underline{\text{FT}}$: $\quad X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$

$\underline{\text{IFT}}$: $\quad x(t) = \int_{-\infty}^{\infty} X(f)\, e^{j2\pi f t}\, df$

- most noteworthy : 2π factors are gone in many cases (Hz)

Will post a table of FT ~~expressed~~ in Hz.