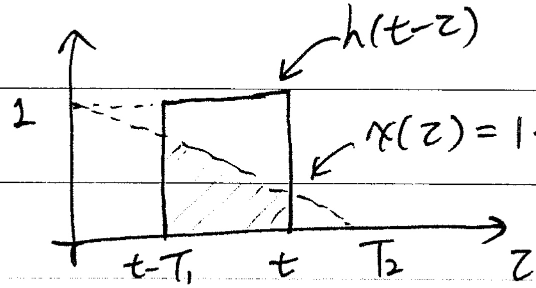
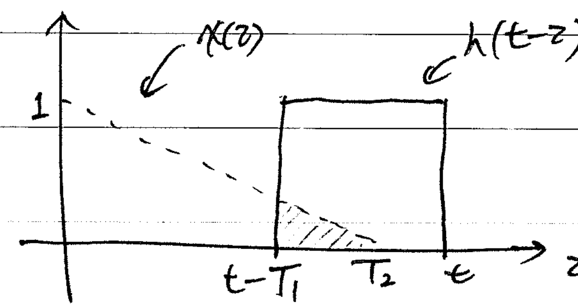
e.g.2) Ramp-Down Triangle (continue)

- for $\begin{pmatrix} t - T_1 > 0 \\ t < T_2 \end{pmatrix} \Leftrightarrow T_1 < t < T_2$ (full overlap)



$$y(t) = \int_{t-T_1}^{t} \left(1 - \frac{\tau}{T_2}\right) d\tau = (\text{skip calc.}) = -\frac{T_1}{T_2} t + \left(T_1 + \frac{T_1^2}{2T_2}\right)$$

- for $\begin{pmatrix} t > T_2 \\ t - T_1 < T_2 \end{pmatrix} \Leftrightarrow T_2 < t < T_1 + T_2$ (partial overlap)

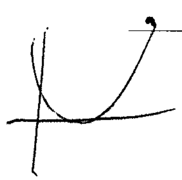


$$y(t) = \int_{t-T_1}^{T_2} \left(1 - \frac{\tau}{T_2}\right) d\tau \quad (\text{skip calc.})$$

$$= \frac{t^2}{2T_2} - \left(1 + \frac{T_1}{T_2}\right) t + \left(T_1 + T_2 - \frac{T_2^2 - T_1^2}{2T_2}\right)$$

↓

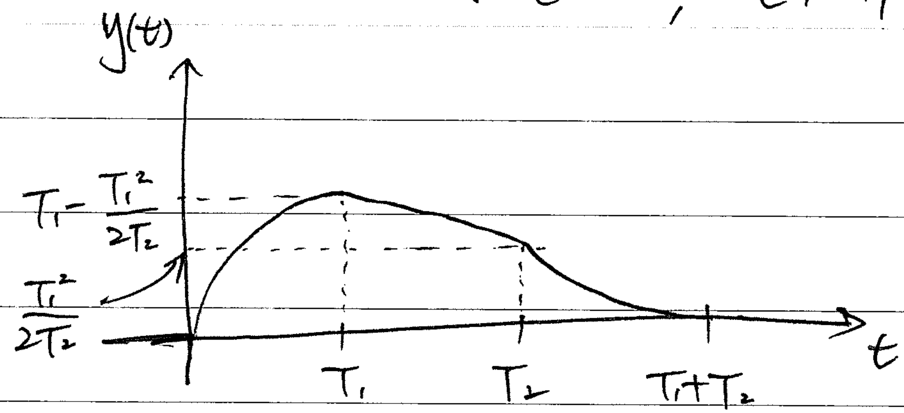
concave upwards

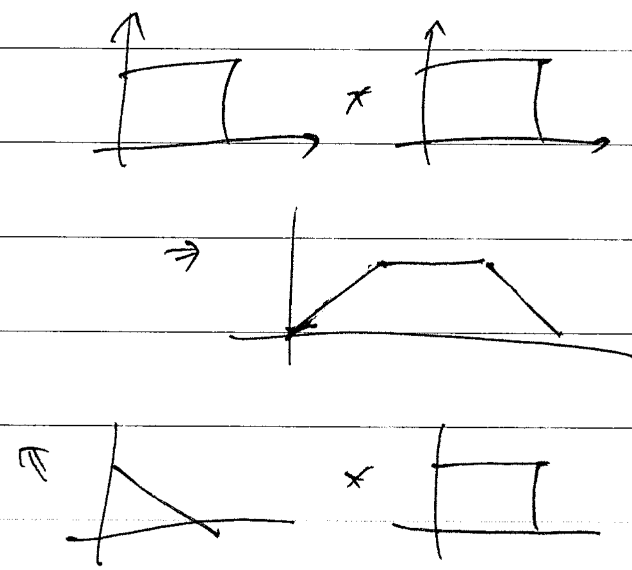


- for $t - T_1 > T_2$, $y(t) = 0$ (no overlap)

◦ Final answer :

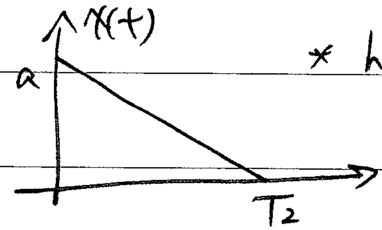
$$y(t) = \begin{cases} 0 \ , & t < 0 \\ -\dfrac{t^2}{2T_2} + t \ , & 0 < t < T_1 \quad \left(\begin{array}{c}\text{quadratic} \\ \text{partial overlap}\end{array}\right) \\ -\dfrac{T_1}{T_2} t + \left(T_1 + \dfrac{T_1^2}{2T_2}\right) , & T_1 < t < T_2 \quad \left(\begin{array}{c}\text{linear} \\ \text{full overlap}\end{array}\right) \\ \dfrac{t^2}{2T_2} - \left(\dfrac{T_1+T_2}{T_2}\right) t + \dfrac{(T_1+T_2)^2}{2T_2} \ , & T_2 < t < T_1 + T_2 \quad \left(\begin{array}{c}\text{quadratic} \\ \text{partial overlap}\end{array}\right) \\ 0 \ , & t > T_1 + T_2 \end{cases}$$





$*$ Denote $y(t)$ on previous slide as $y_0(t)$

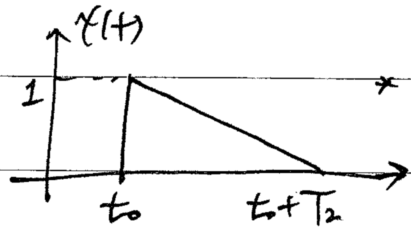
- What is output $y(t)$ when input is:



$* \, h(t) \Rightarrow y(t) = a\, y_0(t)$

since it's linear!

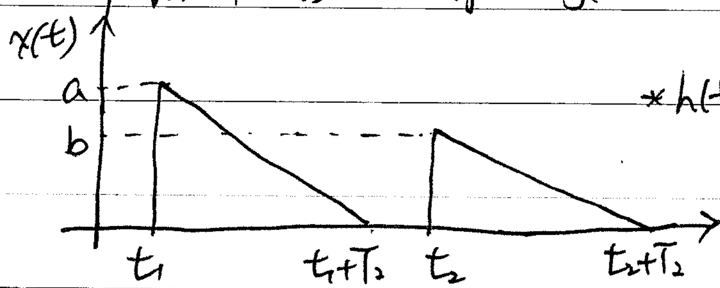
- What is output $y(t)$ when input is:



$* \, h(t) \Rightarrow y(t) = y_0(t - t_0)$

since it's time-invariant!

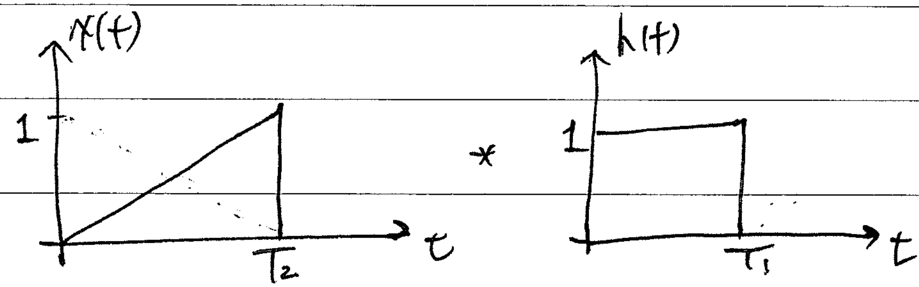
- What is output $y(t)$ when input is:



$* \, h(t) \Rightarrow y(t) = a\, y_0(t - t_1) + b\, y_0(t - t_2)$

since it's LTI!

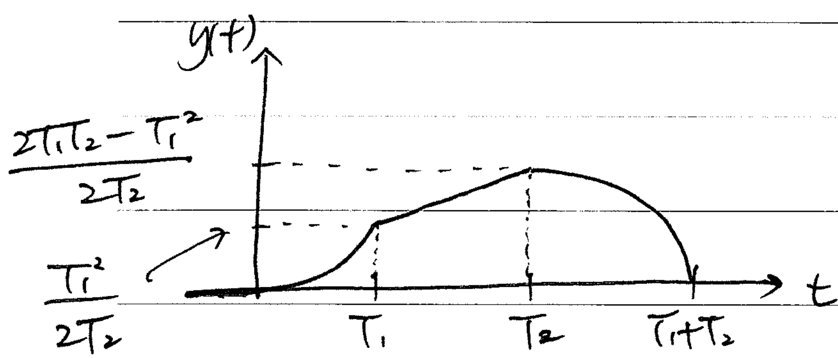
e.g. 3) "Ramp-Up Triangle" (Do-It-Yourself)



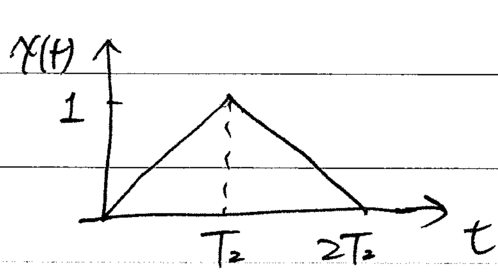
$= \quad y(t) = ?$

$(T_2 > T_1)$

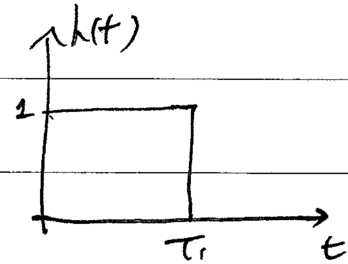
$$
y(t) = \begin{cases}
0, & t < 0 \\
\dfrac{t^2}{2T_2}, & 0 < t < T_1 \\
\dfrac{T_1}{T_2}t - \dfrac{T_1^2}{2T_2}, & T_1 < t < T_2 \\
-\dfrac{1}{2T_2}t^2 + \dfrac{T_1}{T_2}t + \dfrac{T_2^2 - T_1^2}{2T_2}, & T_2 < t < T_1 + T_2 \\
0, & t > T_1 + T_2
\end{cases}
$$



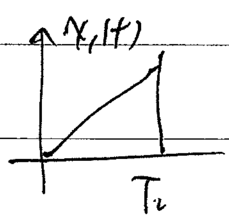
- Now, you could combine both convolution examples to find the conv. of a rectangle with a triangle without much work invoking linearity and time-invariance.



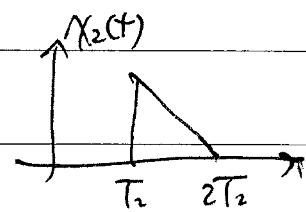
$*$



$= y(t) = ?$



$+$



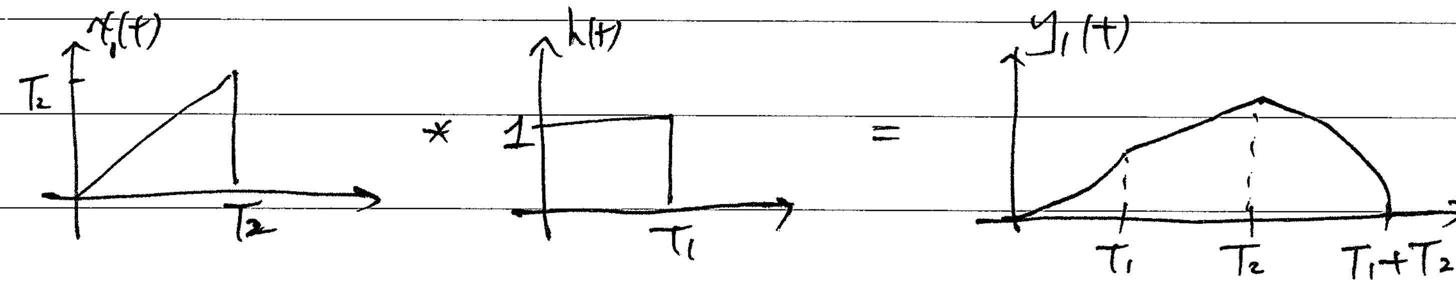
$\downarrow$ $\qquad\qquad$ $\downarrow$

~~$y(t)$~~ $y_1(t)$ $\qquad\qquad$ $y_2(t)$

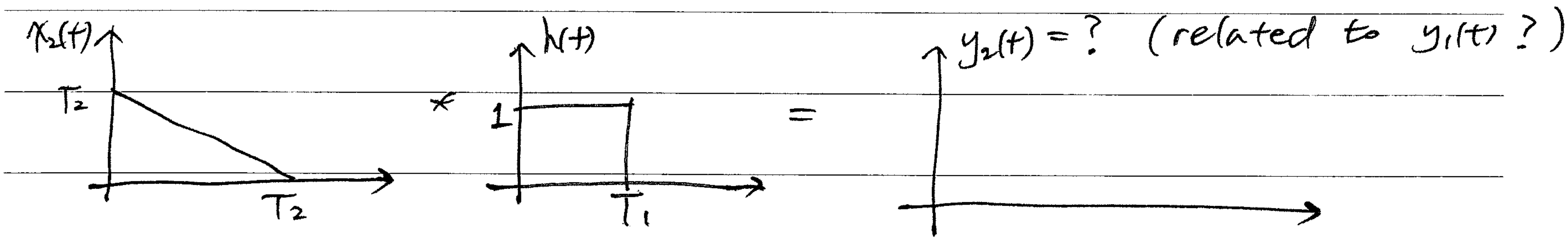
$$\underline{y(t) = y_1(t) + y_2(t - T_2)}$$

* Observations on Convolution of Rectangle with Ramp-Up / Ramp-Down Triangle

- Given the following convolution:



Can we find the conv. below without having to graphically work through the conv. procedure?



$\Rightarrow$ <u>observations</u>

$x_2(t) = x_1(-(t-T_2))$

$h(t) = h(-(t-T_1))$

Since $y_1(t) = x_1(t) * h(t)$, $y_1(-t) = x_1(-t) * h(-t)$

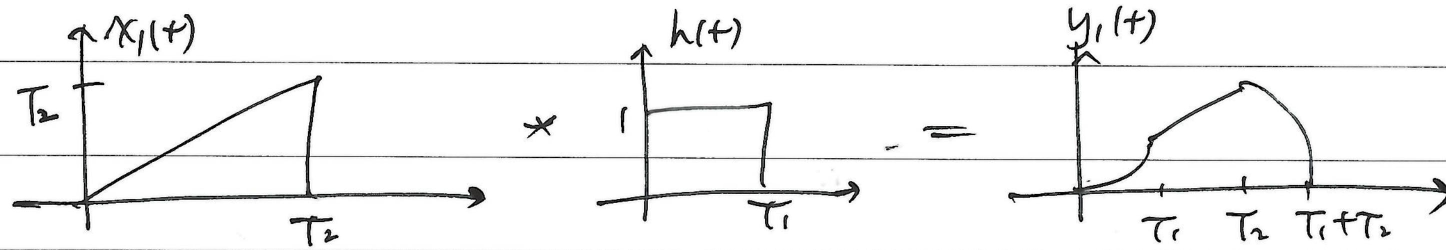
Now $y_2(t) = x_2(t) * h(t)$

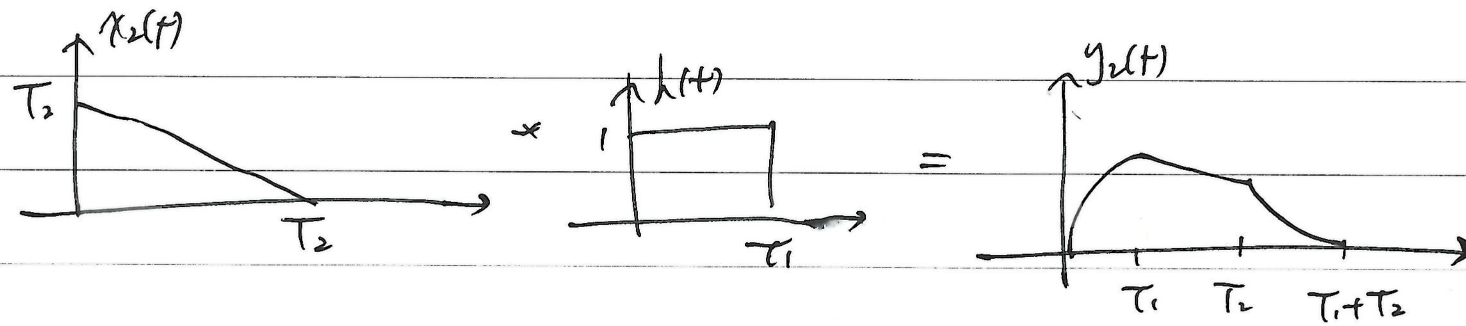
$= x_1(-(t-T_2)) * h(-(t-T_1))$

Compare

Thus, $y_2(t) = y_1(-(t-(T_1+T_2)))$

∘ $y_2(t)$ is $y_1(t)$ flipped (time-reversed) and then shifted to the right to start at $t=0$.





//