ECE 495N, Fall'09 GRIS 280, MWF 1130A – 1220P

HW#9: Due Wednesday Dec.9 in class. This is the last HW for the semester.

Pauli spin matrices: **(2x2) Identity matrix:**

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sigma_y = \begin{bmatrix} 0 & -i \\ +i & 0 \end{bmatrix}, \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

1. What are the eigenvalues of the (2x2) matrix

$$\vec{\sigma}.\hat{n} \equiv \sigma_x \sin\theta\cos\phi + \sigma_y \sin\theta\sin\phi + \sigma_z \cos\theta$$

Show that the corresponding eigenvectors can be written as $\begin{Bmatrix} c \\ s \end{Bmatrix}, \begin{Bmatrix} -s^* \\ c^* \end{Bmatrix}$,

where $c \equiv \cos\frac{\theta}{2}\, e^{-i\varphi/2}$, $s \equiv \sin\frac{\theta}{2}\, e^{+i\varphi/2}$

2. Consider a device with two spin-degenerate levels described by $[H] = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix}$. It is connected to four magnetic contacts with one pointing along $+\hat{z}$ and one along $-\hat{z}$ described by $[\Sigma_1] = -\frac{i}{2}\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}$, $[\Sigma_2] = -\frac{i}{2}\begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}$

where α and β are real numbers with say, $\alpha > \beta$. The other two are identical contacts but one points along $+\hat{n}$ and one along $-\hat{n}$ where $\hat{n} = \hat{z}\cos\theta + \hat{x}\sin\theta$.

a. Show that Σ_1 and Σ_2 can be written in the form $-i(aI + b\sigma_z)/2$ and $-i(aI - b\sigma_z)/2$ respectively and obtain a and b.

b. Show that for the other two contacts we can write

$$\Sigma_3 = \frac{-i}{2}(aI + b\cos\theta\, \sigma_z + b\sin\theta\, \sigma_x)$$

and $$\Sigma_4 = \frac{-i}{2}(aI - b\cos\theta\, \sigma_z - b\sin\theta\, \sigma_x)$$ respectively.

c. For such a multiterminal structure the current is given by

$$I_m(E) = \frac{q}{h}\sum_j T_{mn}(f_m - f_n) \qquad \text{with} \qquad T_{mn} = Trace(\Gamma_m G \Gamma_n G^+)$$

which is a generalization of the two-terminal result we discussed before:

$$I(E) = \frac{q}{h} Trace(\Gamma_1 G \Gamma_2 G^+)(f_1 - f_2)$$

Find the transmission from contact 1 ($+\hat{z}$) to each of the the other three contacts, that is, T_{21}, T_{31} and T_{41}.

Note: $G = [EI - H - \Sigma_1 - \Sigma_2 - \Sigma_3 - \Sigma_4]^{-1}$

and $\Gamma_m = i\left[\Sigma_m - \Sigma_m^+\right]$, m= 1,2,3 and 4.