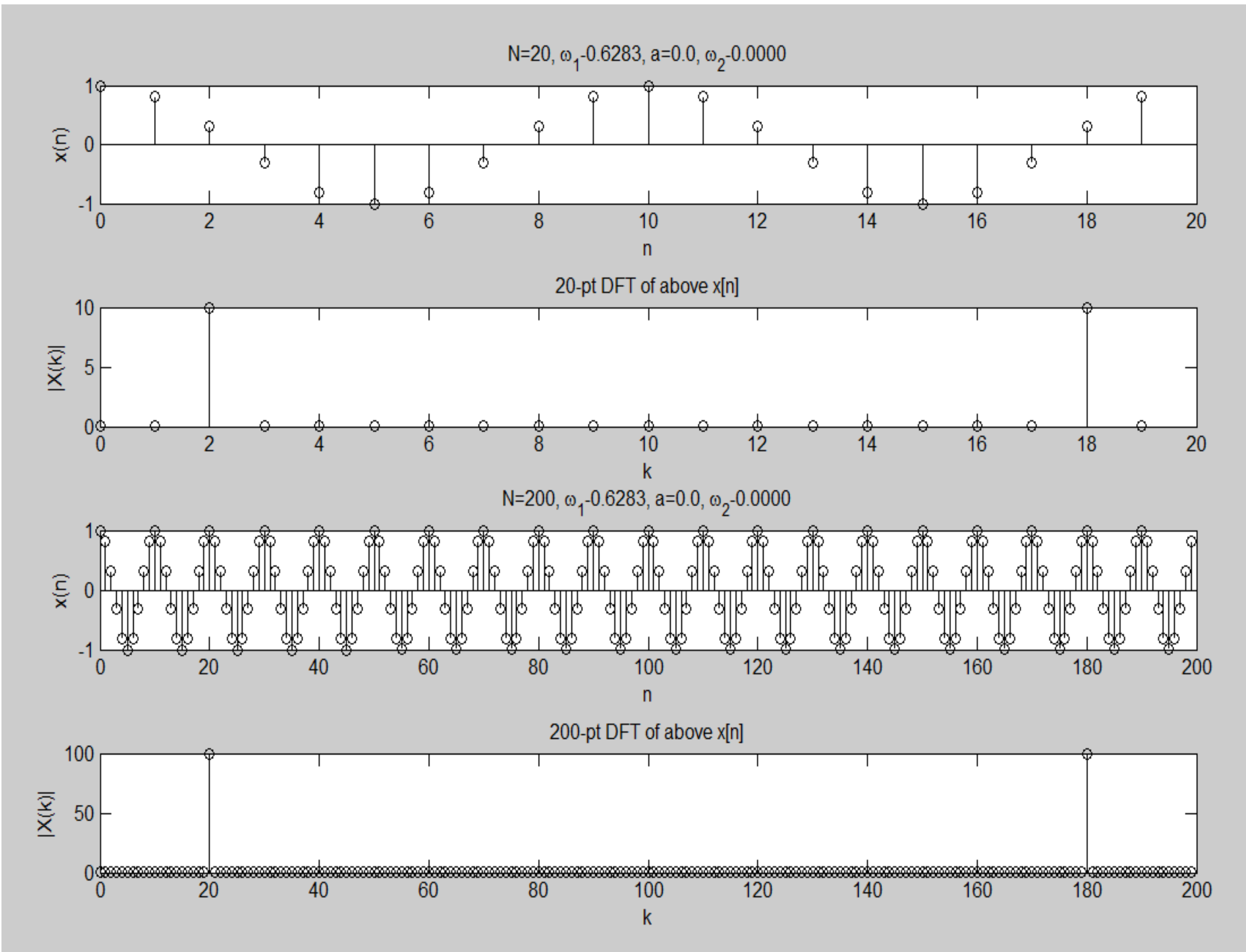
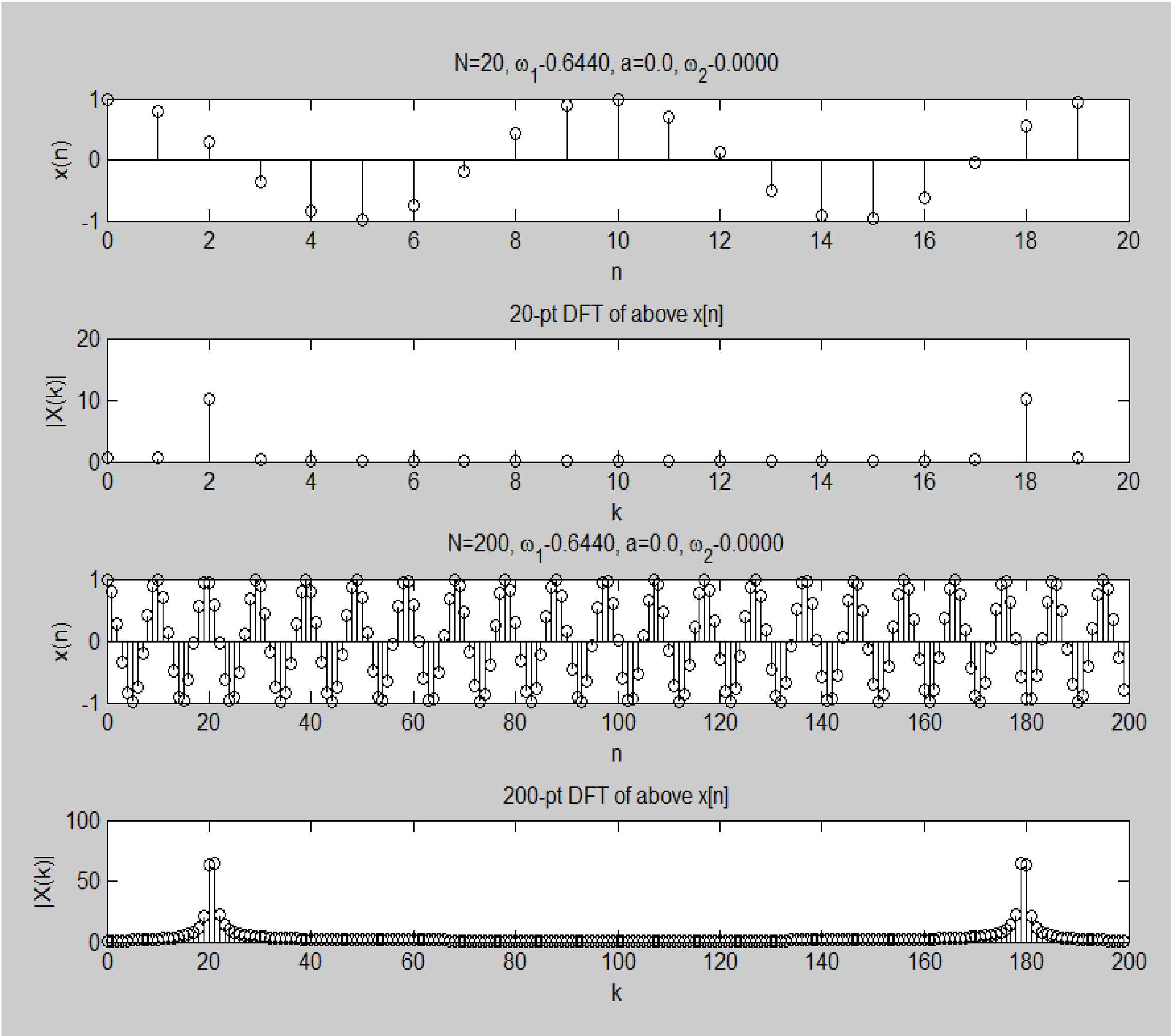
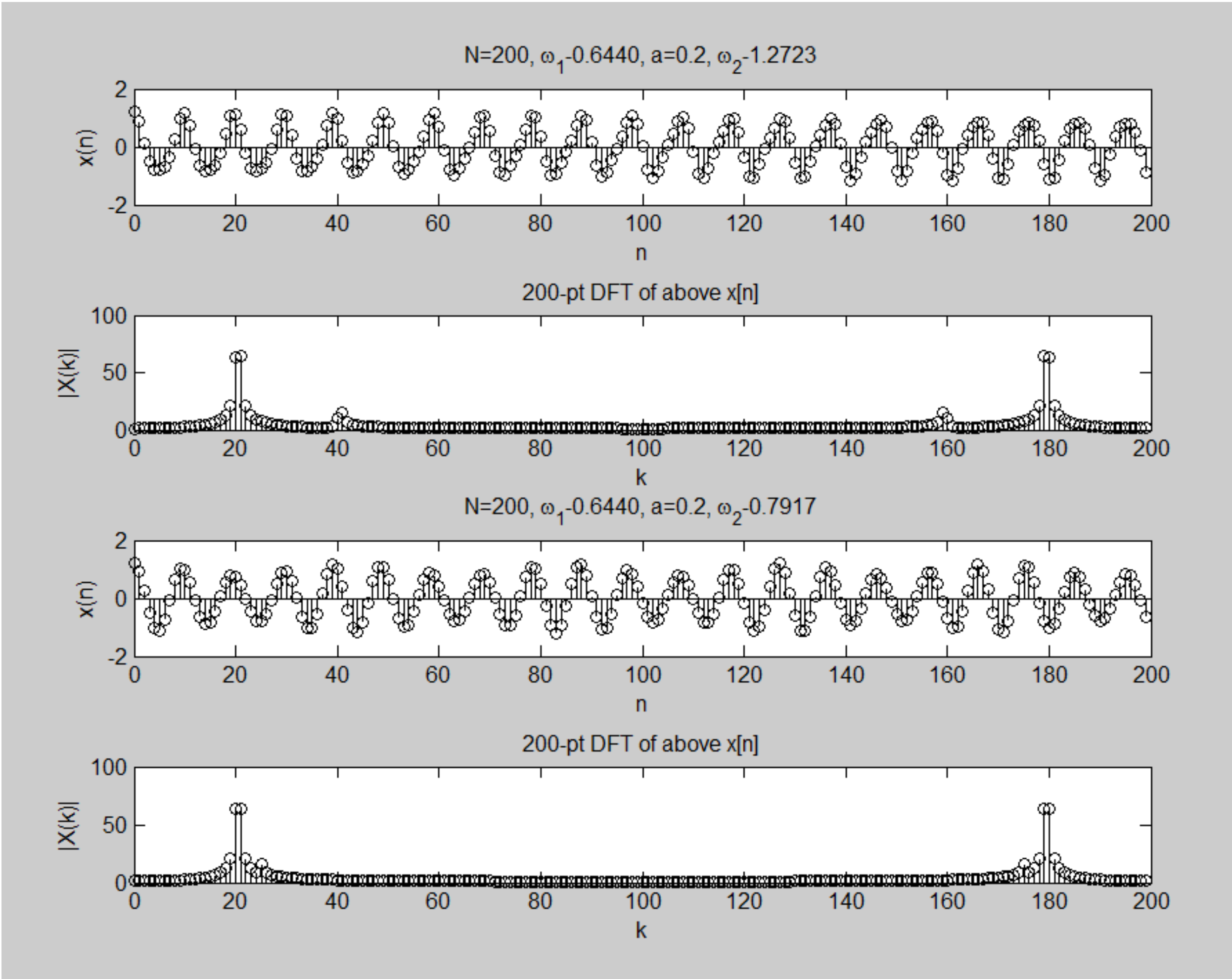
ECE 438 Hw5 Soln




N=20, ω1-0.6440, a=0.0, ω2-0.0000
x(n)
n
20-pt DFT of above x[n]
|X(k)|
k
N=200, ω1-0.6440, a=0.0, ω2-0.0000
x(n)
n
200-pt DFT of above x[n]
|X(k)|
k


N=200, ω1-0.6440, a=0.2, ω2-1.2723
x(n)
n
200-pt DFT of above x[n]
|X(k)|
k
N=200, ω1-0.6440, a=0.2, ω2-0.7917
x(n)
n
200-pt DFT of above x[n]
|X(k)|
k

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%Coding for Hw5 Pr1

N=200*ones(1,8);
N(1)=20;
N(3)=20;
w1=[0.62831853*ones(1,2) 0.64402649*ones(1,4)];
w2=[zeros(1,4) 1.27234502 0.79168135];
a=[zeros(1,4) 0.2*ones(1,2)];
b=zeros(1,6);

for k=1:6
    figure(floor((k+1)/2))
    n=[0:N(k)-1];
    x=cos(w1(k)*n)+a(k)*cos(w2(k)*n)+b(k)*randn(1,N(k));
    X=fft(x);

    subplot(4, 1, 2*mod((k-1),2)+1)
    stem(n,x,'k')
    xlabel('n')
    ylabel('x(n)')
    title(sprintf('N=%d, \\omega_1-%0.4f, a=%0.1f, \\omega_2-%0.4f',...
        N(k), w1(k), a(k), w2(k)));

    subplot(4,1,2*mod(k-1,2)+2)
    stem(n, abs(X),'k')
    xlabel('k')
    ylabel('|X(k)|')
    title(sprintf('%d-pt DFT of above x[n]', N(k)))
    orient('tall')
end
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1.C. The significance of each case.

Case 1 & 2: The first cosines has $w_1 = 0.62831853 = \frac{2\pi}{10} = 2\pi(\frac{2}{20})$

Thus, peaks occur at $\begin{cases} k=2 \text{ and } k = 20-2 = 18 \text{ for } N=20 \\ k=20 \text{ and } K = 200-20 = 180 \text{ for } N=200 \end{cases}$

Case 3 & 4: $w_1 = 0.64402649 = 2\pi(\frac{2.05}{20}) = 2\pi \cdot (\frac{20.5}{200})$

2.05 and 20.5 are not integers, instead of having single peak, we can see the shape of shifted $\left|\frac{\sin(Nw/2)}{\sin(w/2)}\right| e^{-jw(N}$

Case 5, Additional $w_2 = 1.27234502 = 2\pi(\frac{40.5}{200})$

There are also leakage around 20, 40, 200-40, 200-20 peaks because of non-integers.

Case 6: $w_2 = 0.79168135 = 2\pi(\frac{25.2}{200})$

two peaks at N=20 and 25 are emerged from figure

2. a. N-point DFT of $x(n)$

$$X^{(N)}(k) = \sum_{n=0}^{N} x(n) e^{-j\frac{2\pi k}{N}n}$$

$$= \sum_{m=0}^{\frac{N}{2}-1} x(2m) e^{-j\frac{2\pi k}{N}2m} + \sum_{l=0}^{\frac{N}{2}-1} x(2l+1) e^{-j\frac{2\pi k}{N}(2l+1)}$$

Let $x_0(m) = x(2m) \quad m = 0, \cdots, \frac{N}{2}-1$

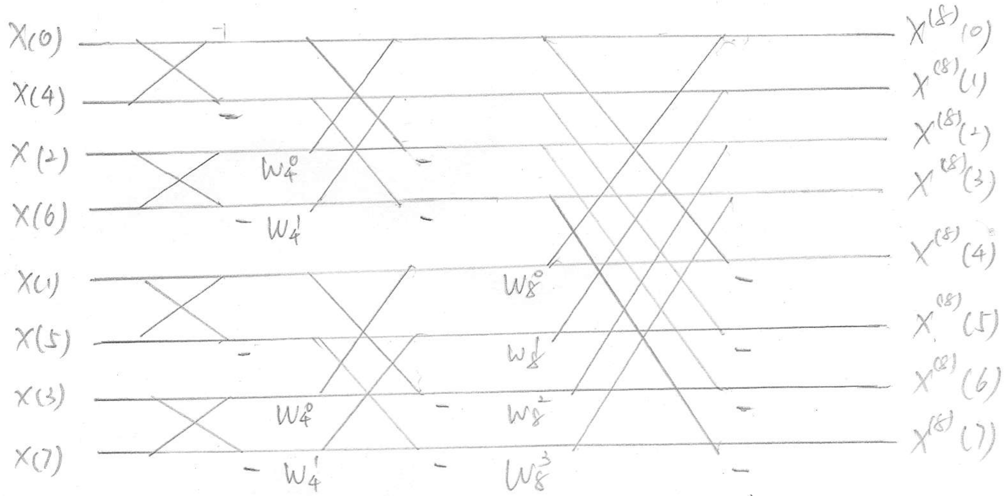
$x_1(l) = x(2l+1) \quad l = 0, \cdots, \frac{N}{2}-1$

$$X^{(N)}(k) = \sum_{m=0}^{\frac{N}{2}-1} x_0(m) e^{-j\frac{2\pi k}{\frac{N}{2}}m} + e^{-j\frac{2\pi k}{N}} \sum_{l=0}^{\frac{N}{2}-1} x_1(l) e^{-j\frac{2\pi k}{\frac{N}{2}}l}$$

$$= X_0^{(\frac{N}{2})}(k) + e^{-j\frac{2\pi k}{N}} X_1^{(\frac{N}{2})}(k)$$

when $N = 8$

$$X^{(8)}(k) = X_0^{(4)}(k) + e^{-j\frac{2\pi k}{8}} X_1^{(4)}(k)$$



b.

DFT $\quad 8^2 = 64$

FFT $\quad 8$

3. a. Decimate by 3, then by 2, then compute 2-point DFT.

① Decimate by 3.

$$X^{(12)}(k) = \sum_{n=0}^{11} x(n)\, e^{-j\frac{2\pi k n}{12}}$$

$$= \sum_{l=0}^{2} \sum_{n=0}^{\frac{12}{3}-1} x(3n+l)\, e^{-j\frac{2\pi k(3n+l)}{12}}$$

$$= \sum_{l=0}^{2} e^{-j\frac{2\pi k l}{12}} \sum_{n=0}^{\frac{12}{3}-1} x(3n+l)\, e^{-j\frac{2\pi k n}{\frac{12}{3}}}$$

$$= X_0^{(4)}(k) + e^{-j\frac{2\pi k}{12}} X_1^{(4)}(k) + e^{-j\frac{4\pi k}{12}} X_2^{(4)}(k)$$

② Decimate by 2.

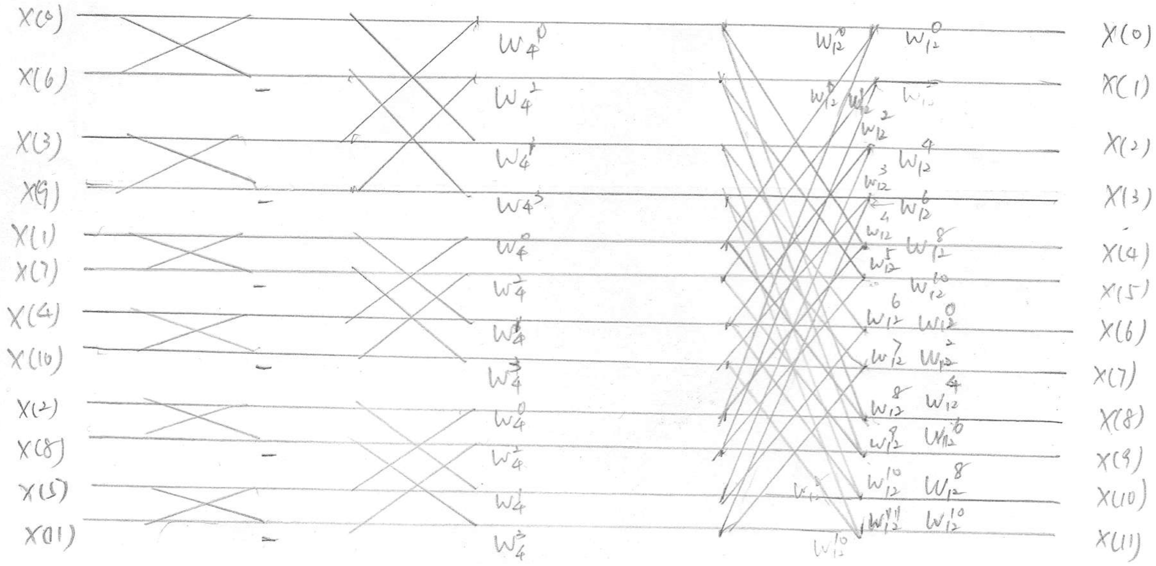
$$X^{(4)}(k) = X_0'^{(2)}(k) + e^{-j\frac{2\pi k}{4}} X_1'^{(2)}(k)$$

③ 2-point DFT for X_0', X_1'

$$X^{(2)}(k) = \sum_{n=0}^{1} x(n)\, e^{-j\pi k n}$$

$$\Rightarrow X^{(2)}(0) = X(0) + X(1)$$

$$X^{(2)}(1) = X(0) - X(1)$$



b. DFT $12^2 = 144$

FFT $3 \times 4 + 2 \times 3 \times 4 = 36$

4. Inverse DFT.

$$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k n}{N}}$$

$$= \frac{1}{N}\left(\left(\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi k n}{N}}\right)^*\right)^*$$

$$= \frac{1}{N}\left(\sum_{k=0}^{N-1} X^*(k)\, e^{-j\frac{2\pi k n}{N}}\right)^*$$

① preprocessing - take complex conjugate of $X(k)$ to get $X^*(k)$

② Forward DFT on $X^*(k) \longrightarrow \text{DFT}\{X^*(k)\}$

③ Post processing - take complex conjugate of $\text{DFT}\{X^*(k)\}$ and divide by N.

5. a. $5120 = 5 \times 1024 = 5 \times 2^{10}$

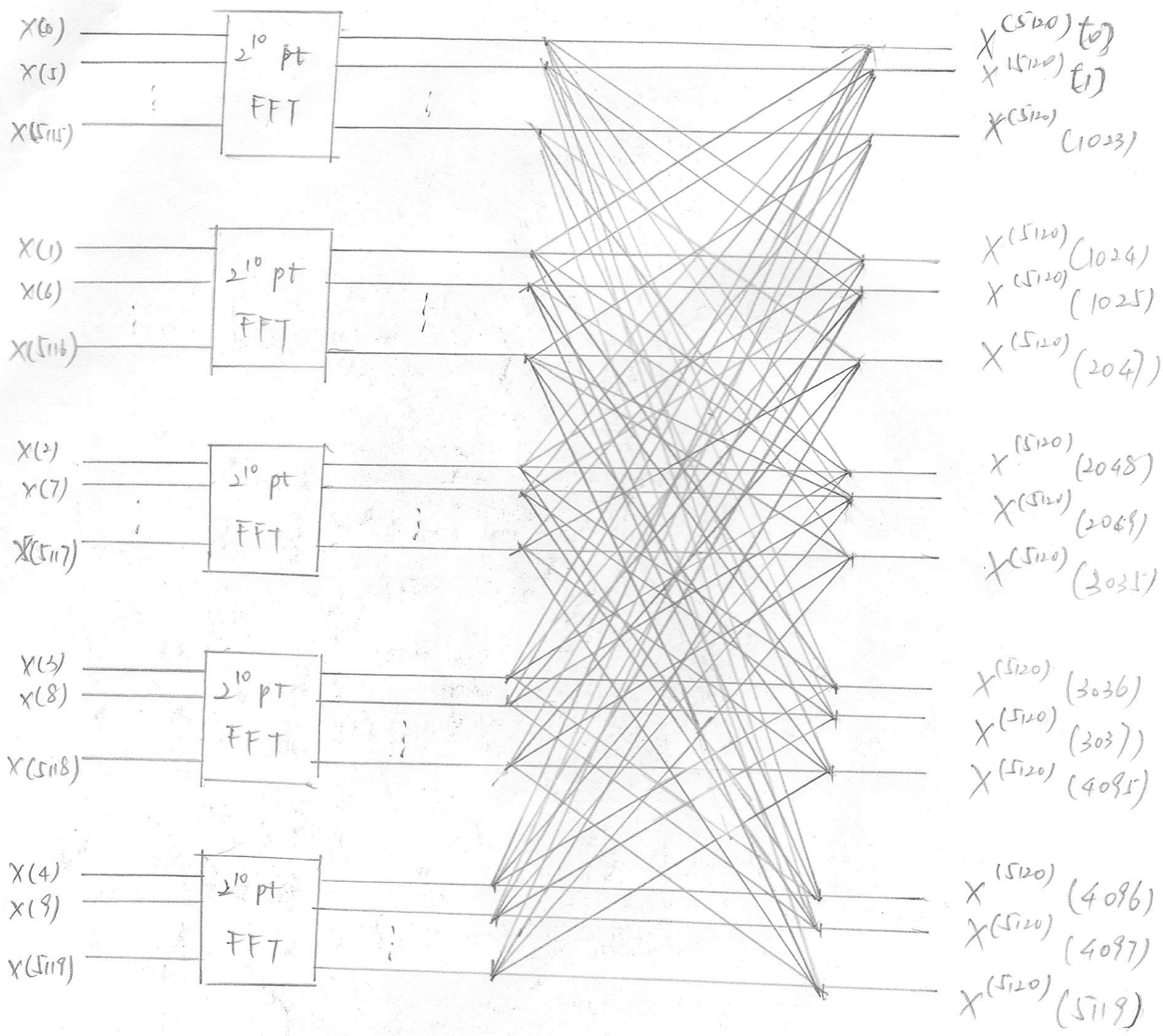
$$X^{(5120)}(k) = \sum_{l=0}^{5-1}\sum_{n=0}^{2^{10}-1} x(5n+l)\, e^{-j\frac{2\pi k(5n+l)}{5120}}$$

$$= \sum_{l=0}^{4} e^{-j\frac{2\pi k l}{5120}} \sum_{n=0}^{1023} x(5n+l)\, e^{-j\frac{2\pi k n}{1024}}$$

$$= \sum_{l=0}^{4} W_{5120}^{kl}\, X_l^{(1024)}(k)$$

Use FFT to calculate $X_l^{(1024)}$, then combine with W_{5120}^{kl}

b.



c. From the diagram above, the number of complex multiplication is $4\cdot(5120) + 5(10-1)\frac{1024}{2} = 43520$

DFT. $(5120)^2 = 26214400$