ECE 301: Homework 3

Due: 7/7/15

Warm-Up Problems

Problem 1

Consider two periodic signals, $x_1(t)$ and $x_2(t)$. Signal $x_1(t)$ has period 2 and Fourier Series coefficients $a_1 = a_{-1} = 2$ and all other $a_k = 0$. Signal $x_2(t)$ has period 3 and Fourier Series coefficients $a_1 = j$, $a_{-1} = -j$ and all other $a_k = 1$.

Part a:

Plot $x_1(t)$ and $x_2(t)$.

Part b:

Let $y(t) = x_1(t) + x_2(t)$. Find the Fourier Series coefficients of $y(t)$.

Problem 2

Given the Fourier Series coefficients of a continuous-time signal that is periodic with period 4, find the signal, $x(t)$.

Part a:

$a_k = \delta[k-3] + \delta[k+3]$

Part b:

$a_k = e^{-2|k|}$

MATLAB Portion

Introduction

This section is intended to familiarize you with fourier series. Please review the analysis and synthesis equations for the Continuous-Time Fourier Series (CTFS) before starting. All files necessary are included in the waveforms.zip file.

1 Square Wave

Download square_wave.mat and load it in Matlab (`load square_wave.mat`). It will contain a vector continaing the time, `t`, a vector with the square waveform, `x`, and a variable with the period, `T`.

1.1 Fourier Series Coefficients

Find the formula for the Fourier Series (CTFS) Coefficients, a_k. Next, find an expression for b_k such that:

$$x(t) = b_0 + \sum_{k=1}^{\infty} b_k cos\left(\frac{2\pi}{T}kt\right) \tag{1}$$

Let $\tilde{x}(t)$ be an approximation to $x(t)$ that has only the first few harmonics.

$$\tilde{x}(t) = b_0 + \sum_{k=1}^{N} b_k cos\left(\frac{2\pi}{T}kt\right) \tag{2}$$

1.2 Deliverables

For the first 8 harmonics (not including b_0 and therefore there are 6 total graphs), plot $x(t)$ and $\tilde{x}(t)$ in the same figure (`plot(t,x,t,x_tilde)`).

Make a single plot of $\tilde{x}(t)$ for $N = 100$ (first 100 harmonics) with $x(t)$ in the same axes as before. Note what happens as the number of harmonics is large.

1.3 Checklist

- Expression for a_k.
- Expression for b_k.
- Plots of first five harmonics.
- Plot using first 100 harmonics.

2 Saw-tooth

Download saw_tooth.mat and load it in Matlab (`load saw_tooth.mat`). It will contain a vector continaing the time, `t`, a vector with the square waveform, `x`, and a variable with the period, `T`.

2.1 Fourier Series Coefficients

Find the formula for the Fourier Series (CTFS) Coefficients, a_k. Next, find an expression for b_k such that:

$$x(t) = b_0 + \sum_{k=1}^{\infty} b_k sin\left(\frac{2\pi}{T}kt\right) \tag{3}$$

Let $\tilde{x}(t)$ be an approximation to $x(t)$ that has only the first few harmonics.

$$\tilde{x}(t) = b_0 + \sum_{k=1}^{N} b_k cos\left(\frac{2\pi}{T}kt\right) \tag{4}$$

2.2 Deliverables

For the first 8 harmonics (8 total graphs), plot $x(t)$ and $\tilde{x}(t)$ in the same figure (`plot(t,x,t,x_tilde)`).

Make a single plot of $\tilde{x}(t)$ for $N = 100$ (first 100 harmonics) with $x(t)$ in the same axes as before. Note what happens as the number of harmonics is large.

2.3 Checklist

- Expression for a_k.
- Expression for b_k.
- Plots of first eight harmonics.
- Plot using first 100 harmonics.

3 Report and Code

For the MATLAB section, please attach a typed-up report with all explicit deliverables, explanations you deem necessary, and your code at the end.