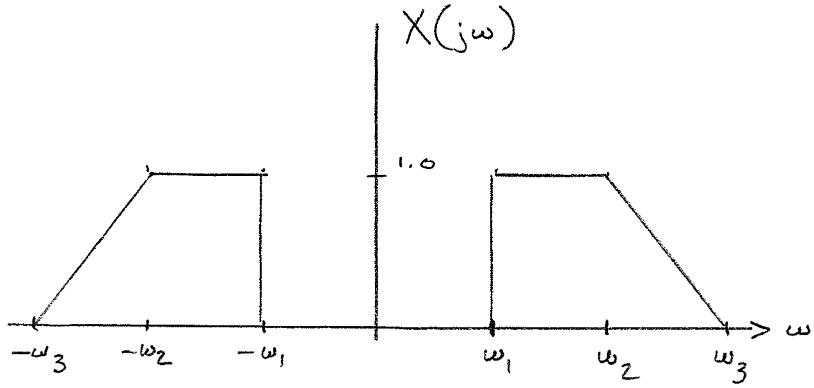
10/23

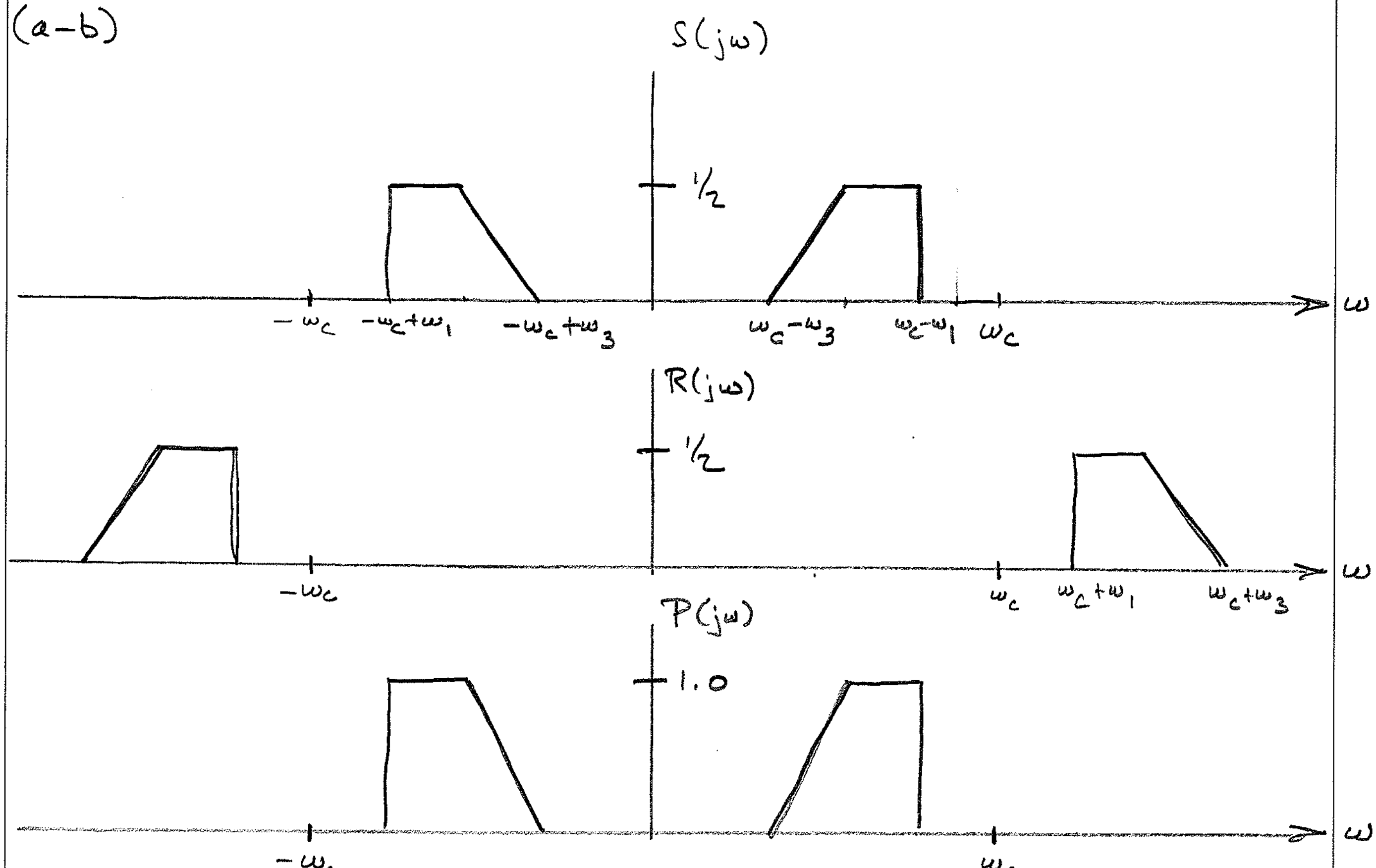
O+W 8.29

Problem shows three schemes for generating SSB. Two are based on filtering to select the desired sideband, one is based on phase shifting. An example message spectrum is given and we are asked to plot the spectra of the SSB modulated signals in all three cases.



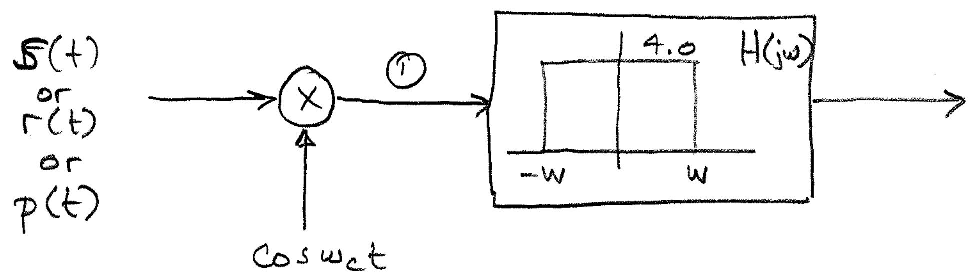
All of these systems were analyzed in some detail in lecture notes. The only change here is in the use of a differently shaped message spectrum $X(j\omega)$.

(a−b)



(freq scales changed from $X(j\omega)$ plot to these).

(c) Show that



works as a demodulator for any of the signals as longs as $W = w_c$ (and Tx, Rx oscillators in-phase)

We did not go through this in lecture, hence should work out in some detail. Note that $r(t)$ and $p(t)$ differ only by scale factor.

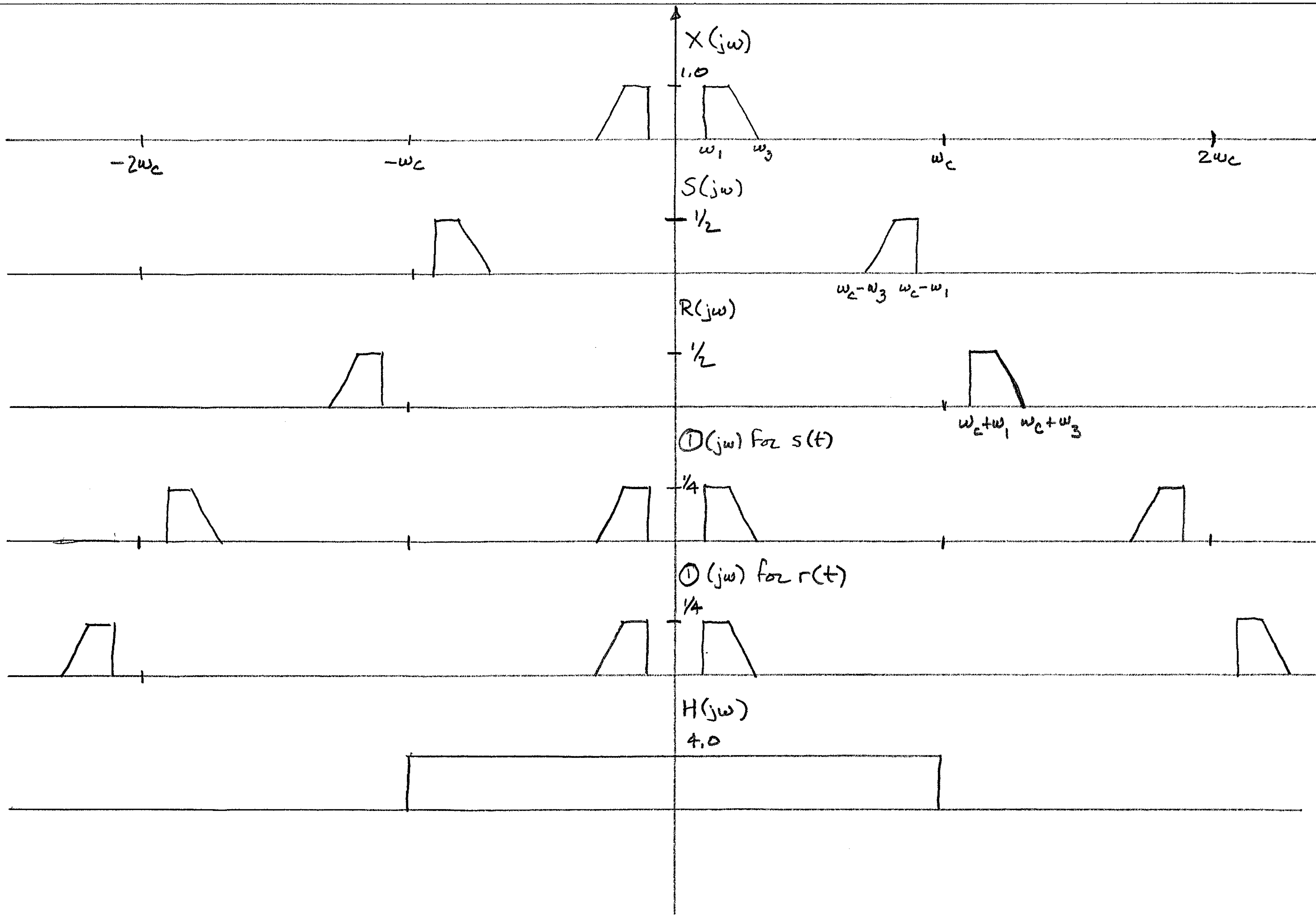
Signal at point 1 is just a DSB-SC wave corresponding to "message" $s(t)$, $r(t)$ or $p(t)$ as the case may be — shift right by w_c, shift left by w_c, scale by $1/2$ gives picture in each case.

See next page for plots.

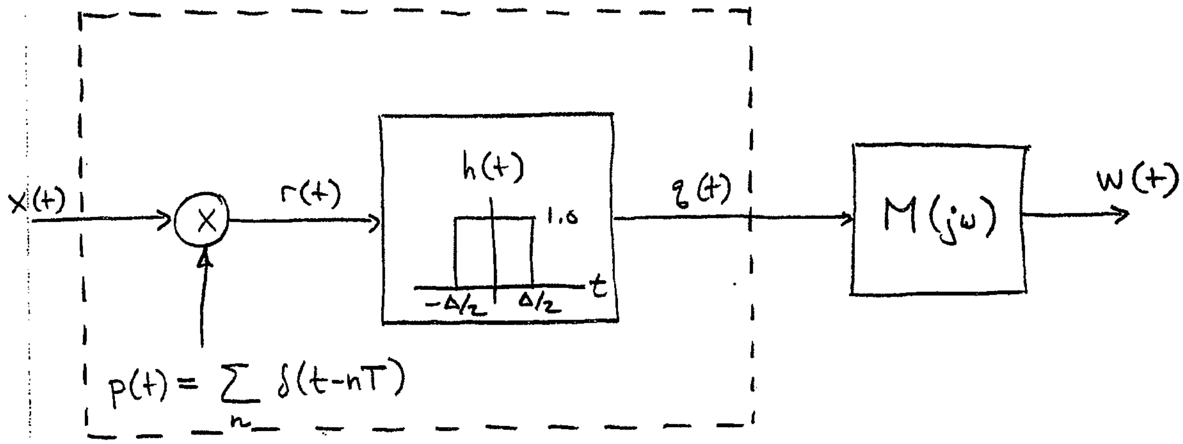
Clear from plot that this demodulator exactly recovers $x(t)$ when input is $r(t)$ or $s(t)$.

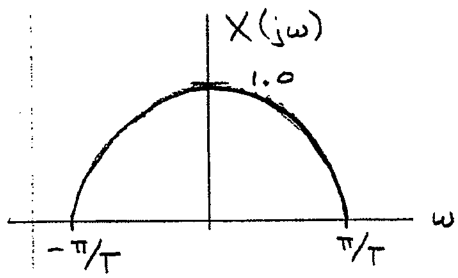
If apply $p(t)$ the demodulator output will be $2\,x(t)$... so could cut gain of $h(t)$ by 50%.

(these gains are really not relevant — the important point is that demod. output is proportional to message)


X(jω)
1.0
ω₁ ω₃
−2ω_c
−ω_c
ω_c
2ω_c
S(jω)
1/2
ω_c−ω₃ ω_c−ω₁
R(jω)
1/2
ω_c+ω₁ ω_c+ω₃
① (jω) for s(t)
1/4
① (jω) for r(t)
1/4
H(jω)
4.0

O + W 8.30





(ie bandlimited as shown).

(a) Find and sketch $R(j\omega)$ and $Q(j\omega)$.

$$r(t) = x(t)\,p(t) = \sum_n x(t)\,\delta(t-nT) = \sum_n x(nT)\,\delta(t-nT)$$

Therefore, from modulation property

$$R(j\omega) = \frac{1}{2\pi} X(j\omega) * P(j\omega)$$

$$= \frac{1}{2\pi} X(j\omega) * \left\{ \frac{2\pi}{T} \sum_k \delta\left(\omega - \frac{2\pi k}{T}\right) \right\}$$

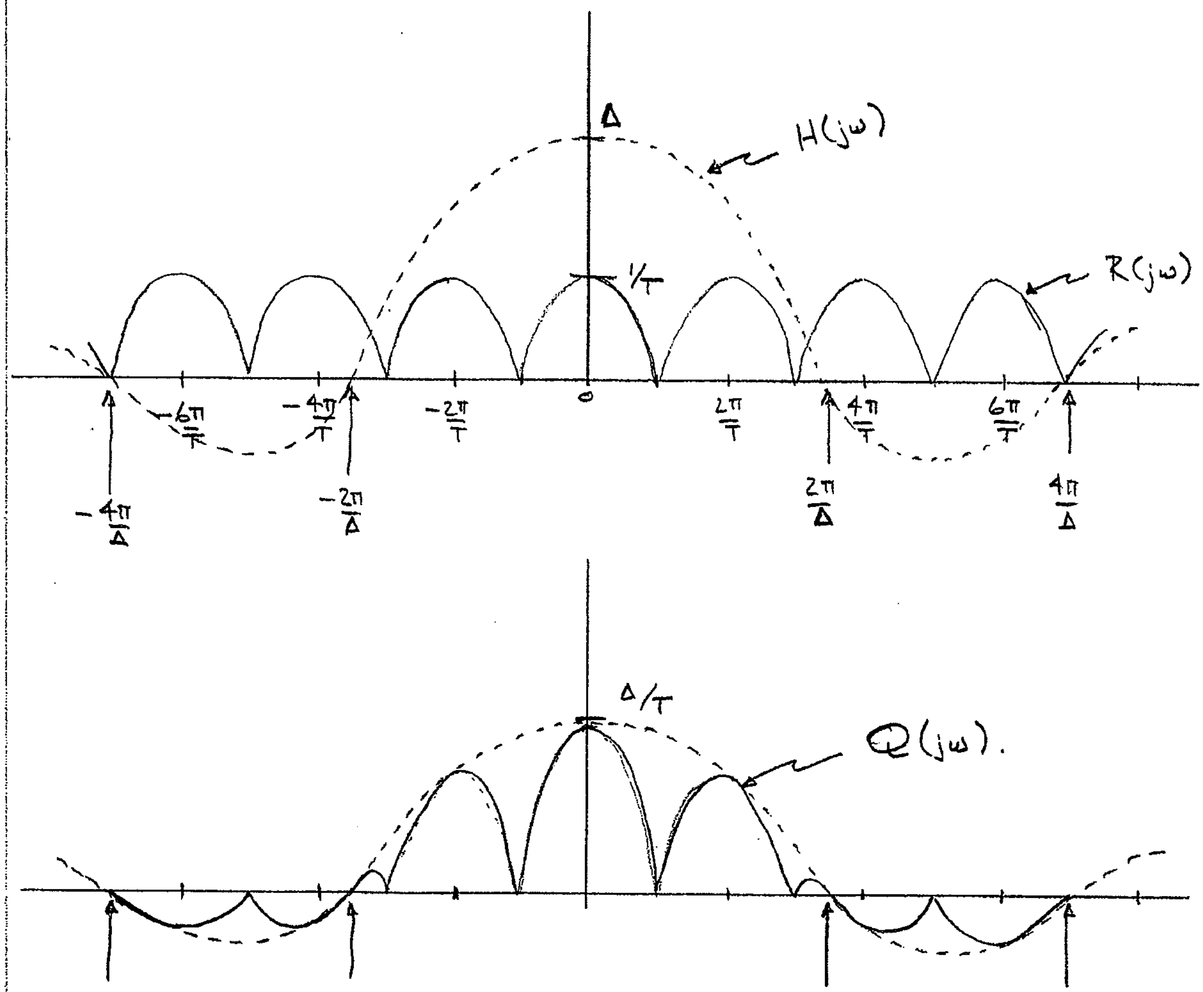
$$= \frac{1}{T} \sum_k X\left(j\left(\omega - \frac{2\pi k}{T}\right)\right)$$

Also

$$h(t) \longleftrightarrow \frac{2\sin(\omega\Delta/2)}{\omega} = \Delta\,\frac{\sin(\omega\Delta/2)}{(\omega\Delta/2)}.$$

and

$$Q(j\omega) = \frac{\Delta}{T}\,\frac{\sin(\omega\Delta/2)}{(\omega\Delta/2)} \sum_k X\left(j\left(\omega - \frac{2\pi k}{T}\right)\right).$$



Picture Drawn for case where 1st zero crossing of sinc is past the bandedge of 1st replication of X spectrum ie

$$\frac{2\pi}{\Delta} > \frac{\pi}{T} \iff 2T > \Delta$$

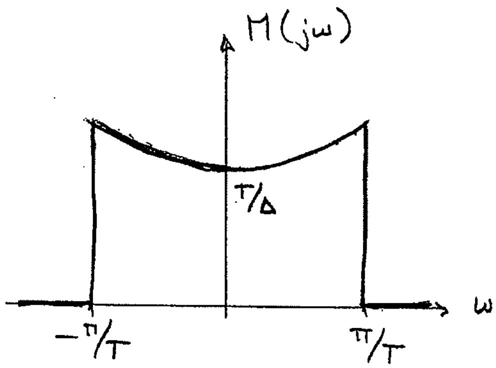
We need this because we need to invert the sinc filter distortion over $-\frac{\pi}{T} < \omega < \frac{\pi}{T}$ and can't build the inverse filter if have a zero in the desired band.

$$\therefore \Delta_{max} = 2T$$

(c) Required compensating filter is (assuming $\Delta < 2T$)

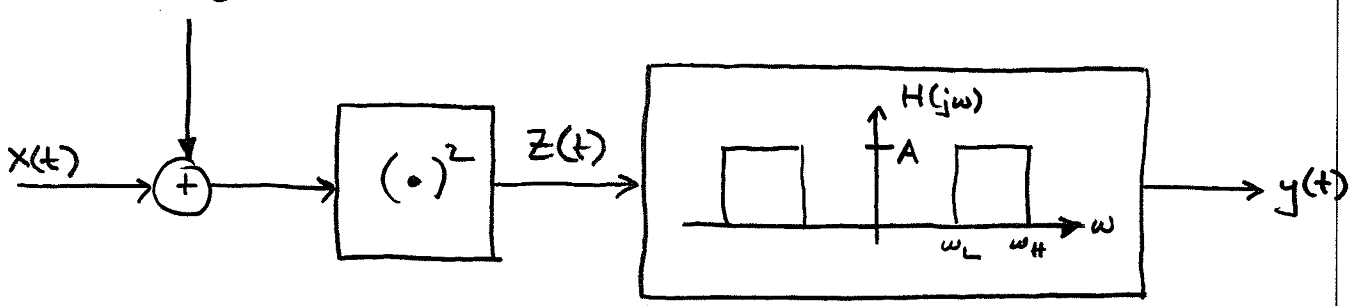
$$M(j\omega) = \begin{cases} T \cdot H^{-1}(j\omega) & |\omega| < \pi/T \\ 0 & \text{else} \end{cases}$$

$$= \begin{cases} \dfrac{T}{\Delta} \dfrac{\omega\Delta/2}{\sin(\omega\Delta/2)} & |\omega| < \pi/T \\ 0 & \text{else.} \end{cases}$$



- smaller Δ makes this filter easier to build.
- small enough Δ, then distortion could be ignored.

O+W 8.34 Modulation without multiplier.



- $X(j\omega)$ bandlimited s.t. $= 0$ for $|\omega| > \omega_M$

Find: A, ω_L, ω_H s.t. $y(t) = x(t)\cos\omega_c t$; any necc. constraints on ω_c and ω_M.

$$z(t) = \left(x(t) + \cos\omega_c t \right)^2 = x^2(t) + 2x(t)\cos\omega_c t + \cos^2\omega_c t$$

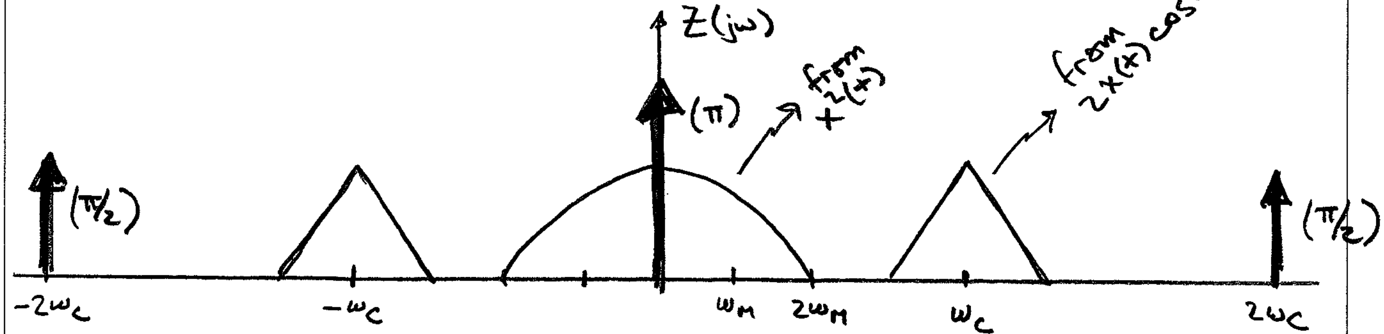
Now to understand the effect of the filter $H(j\omega)$ we must be able to figure out the Fourier Transform of $z(t)$, or at least figure out the spectral bands occupied by the components:

$x^2(t) \longleftrightarrow \frac{1}{2\pi} X(j\omega) * X(j\omega)$ this will be bandlimited but now with twice the bandwidth. ie $= 0$ for $|\omega| > 2\omega_M$

$$2x(t)\cos\omega_c t \longleftrightarrow X(j(\omega-\omega_c)) + X(j(\omega+\omega_c))$$

$$\cos^2\omega_c t = \frac{1+\cos 2\omega_c t}{2} \longleftrightarrow \pi\delta(\omega) + \frac{\pi}{2}\delta(\omega-2\omega_c) + \frac{\pi}{2}\delta(\omega+2\omega_c).$$

$\therefore$ spectrum of $z(t)$ looks roughly like



From the picture we have the following constraint to avoid overlap.

$$2\omega_M < \omega_C - \omega_M \iff 3\omega_M < \omega_C$$

For the remaining parameters we may pick ω_L st.

$$2\omega_M < \omega_L < \omega_C - \omega_M$$

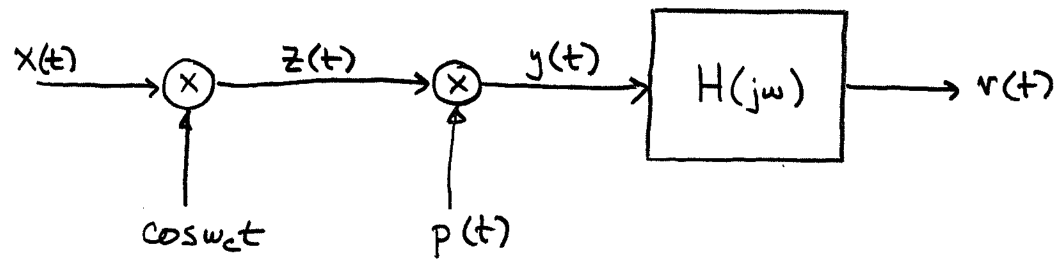
ω_H st.

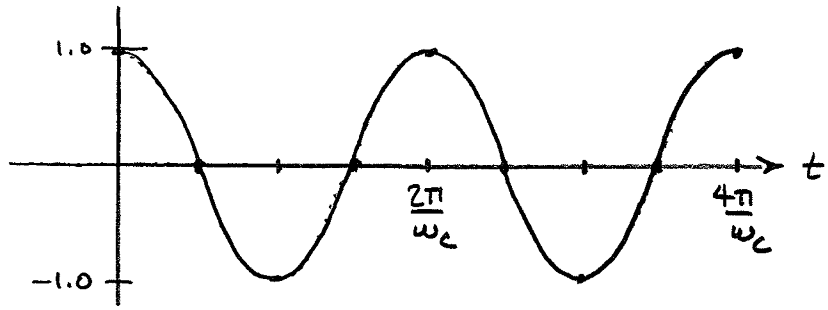
$$\omega_C + \omega_M < \omega_H < 2\omega_C$$

and set

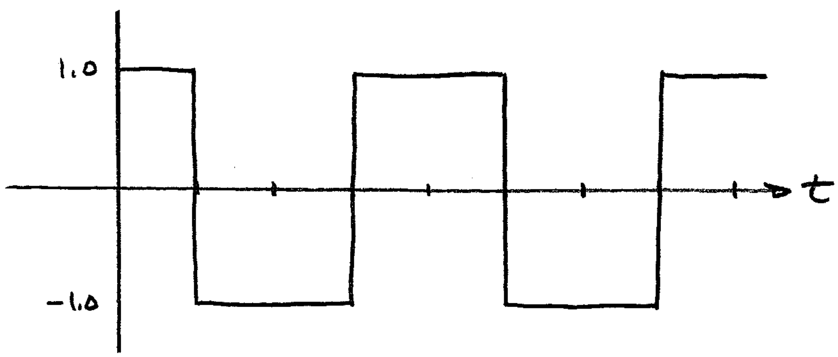
$$A = 1/2.$$

O + W 8.35





$\cos \omega_c t$



$p(t)$

message $x(t)$ is bandlimited to $|\omega| < \omega_M < \omega_c$ as shown in Figure P8.35.

(a) Sketch the real and imaginary parts of $Z(j\omega)$, $P(j\omega)$, $Y(j\omega)$.

First $z(t) = x(t) \cos \omega_c t$ a standard DSC-AM wave hence

$$Z(j\omega) = \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

To find $P(j\omega)$ we must first find the Fourier Series of $p(t)$. We can use Table 4.2 if we don't want to compute it.

$T = \frac{2\pi}{\omega_c}$ (ie $\omega_0 = \omega_c$)

$T_1 = T/4$

$\therefore$ FS. coeffs are

$$a_k = 2\frac{\sin k\omega_c T_1}{k\pi} \qquad \omega_c T_1 = \omega_c \frac{T}{4} = \cancel{\omega_c}\frac{2\pi}{\cancel{\omega_c}}\frac{1}{4}$$

$$= 2\frac{\sin k\pi/2}{k\pi} \quad k \neq 0 \qquad = \pi/2$$

($a_0 = 0$ since average value is zero)

$$\therefore P(j\omega) = 2\pi \sum_{k=-\infty}^{\infty} a_k\, \delta(\omega - k\omega_c)$$

Looking more closely at the a_k see

$$\sin k\pi/2 = \begin{cases} 0 & k = 0, \pm 2, \pm 4 \ldots \\ 1 & k = \ldots -3, 1, 5, 9 \ldots \\ -1 & k = \ldots -1, 3, 7, 11 \ldots \end{cases}$$

Then dividing by $k\pi/2$ and accounting for signs we see

$$2\pi a_k = \begin{cases} 0 & k = 0, \pm 2, \pm 4 \ldots \\ \frac{4}{|k|} & k = \pm 1, \pm 5 \ldots \\ -\frac{4}{|k|} & k = \pm 3, \pm 7 \ldots \end{cases}$$

$y(t) = z(t)\,p(t)$

$$\therefore\ Y(j\omega) = \frac{1}{2\pi} Z(j\omega) * P(j\omega)$$

$$= \frac{1}{2\pi}\, 2\pi \sum_{k \text{ odd}} a_k Z(j(\omega - k\omega_c))$$

coeff in front of $k = \pm 1, \pm 5 \ldots$ terms is

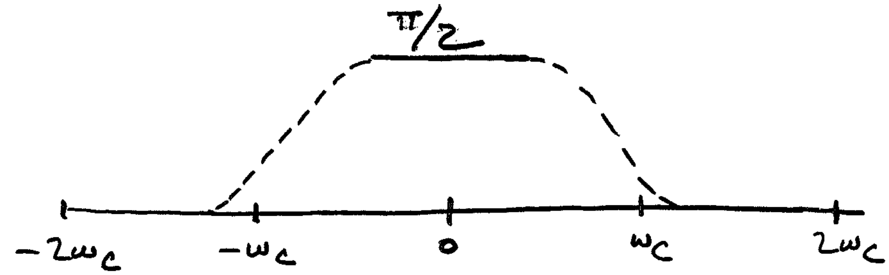
$$\frac{2}{\pi |k|}$$

coeff in front of $k = \pm 3, \pm 7 \ldots$ terms is

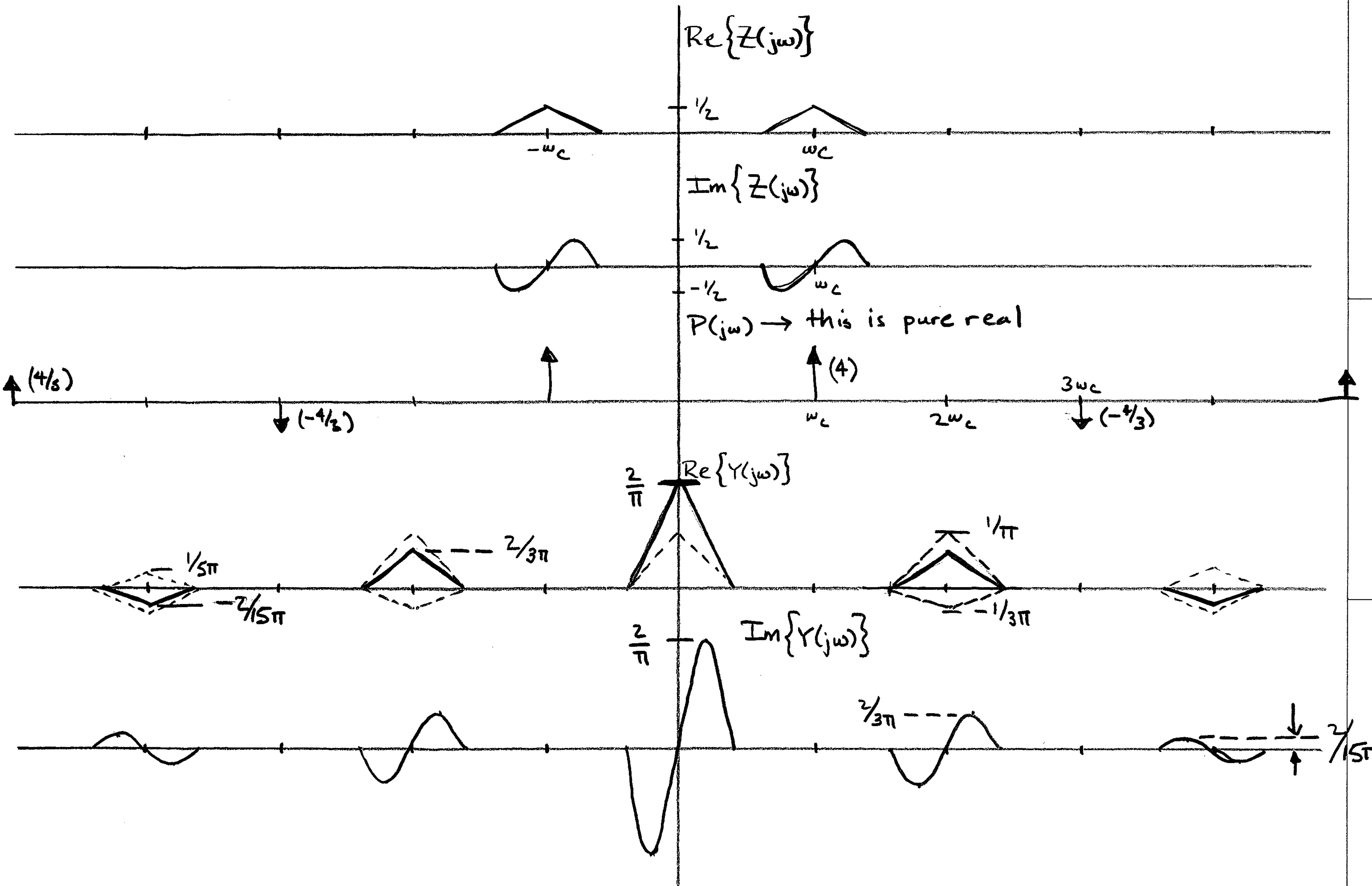
$$-\frac{2}{\pi |k|}$$

See sketches on next page.

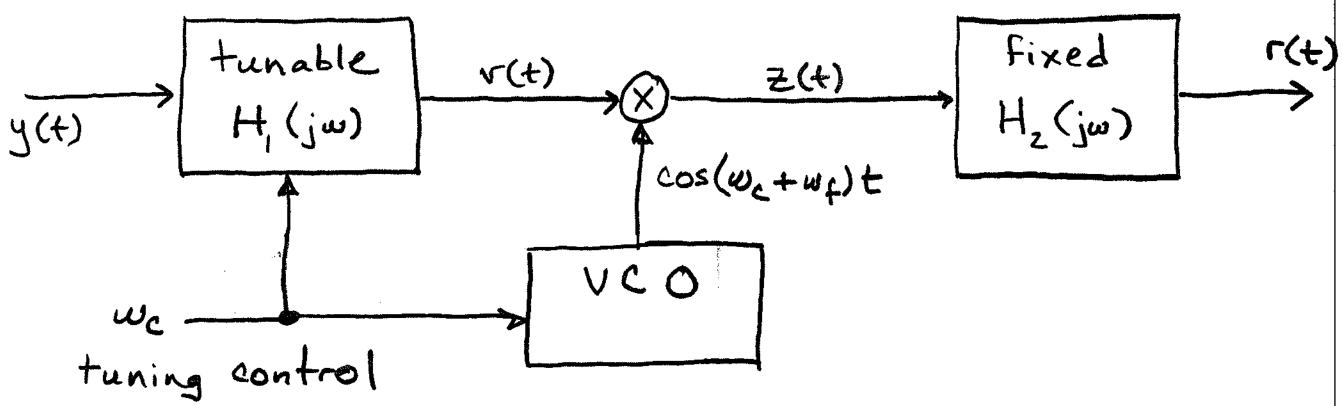
(b) From picture see that a low pass filter with passband gain $A = \pi/2$ and shape



will work to produce $r(t) = x(t)$.


Re{Z(jω)}
1/2
-ω_c
ω_c
Im{Z(jω)}
1/2
-1/2
ω_c
P(jω) → this is pure real
(4/5)
(-4/3)
(4)
ω_c
2ω_c
3ω_c
(-4/3)
2/π
Re{Y(jω)}
1/5π
-2/15π
2/3π
1/π
-1/3π
2/π
Im{Y(jω)}
2/3π
2/15π

O+W 8.36



Goal is to analyze this superheterodyne receiver for the filter shapes given.

(a) Sketch and label $Z(j\omega)$ for $\omega > 0$.

First note that $V(j\omega) = H_1(j\omega)Y(j\omega)$ which has the plot shown on next page.

Now

$$z(t) = v(t)\cos(\omega_c+\omega_f)t$$

$$\updownarrow$$

$$Z(j\omega) = \tfrac{1}{2}V(j(\omega-\omega_c-\omega_f)) + \tfrac{1}{2}V(j(\omega+\omega_c+\omega_f))$$

(see plot on next page).

(b-e) From the plot see that want to pick

$$G = 2/K \qquad \alpha = \omega_f - \omega_M \qquad \beta = \omega_f + \omega_M$$

to just pass the desired term and correct for the gain. We also need to prevent the tails from the course filter from falling into the desired message band ie need

$$\left.\begin{aligned} -\omega_f + \omega_T &< \omega_f - \omega_M \\ \omega_T &< 2\omega_f - \omega_M \end{aligned}\right\} \quad \begin{aligned} \omega_f + \omega_M &< 2\omega_c + \omega_f - \omega_T \\ \omega_T &< 2\omega_c - \omega_M. \end{aligned}$$

Drawn for case of down conversion and $\omega_f < \omega_c$