Midterm Examination 2
ECE 438
Fall 2010
Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete the 4 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.

3. This exam contains 10 pages. Pages 8-9 contain a table of formulas and properties along with the cheat sheet that was created on Rhea prior to the test. The last 2 pages can be used as scratch paper. You may tear out the table, cheat sheet, and the scratch paper **once the exam begins**. If you use a non-trivial fact/property that is not contained in the table or in the cheat sheet, you must justify it (i.e., **prove it**!) in order to get full credit.

4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name:____

Email:____

Signature:_____________________________________

Itemized Scores

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

(15 pts) **1.** Is the following claim true or false? Justify your answer.

Claim: The CSFT of a separable signal $f(x, y)$ is *always* separable.

(15 pts) **2.** What is the CSFT of the signal $f(x, y) = e^{j2\pi x}$? (Justify your answer.)

3. (30 pts) You have a radix 2 FFT subroutine that computes the DFT for 2^M points for any integer value of M. Draw a block diagram for a routine to efficiently compute a 192-point DFT using this radix 2 FFT. (Show the radix 2 FFT subroutine as a black box with no detail regarding what is inside it.) Do not forget to indicate the multiplications by the W_k^n's elsewhere in the diagram. (Note that these were sometimes omitted in the homework solutions for the FFT problems.) However, you do not have to draw *all* the lines in the diagram; use "..." when appropriate (i.e., as long as the pattern is clear).

4. The signal $x[n] = 2\delta[n] + 3\delta[n-1] + 4\delta[n-2]$ is the input of an LTI system with unit impulse response $h[n] = \delta[n] + 5\delta[n-1]$.
(20 pts) a) Compute the 6-point circular convolution of $x[n]$ and $h[n]$ using the definition of the circular convolution (i.e., the summation formula).

Question 4, continued.

(10 pts) b) Is there a relationship between your answer in a) and the output $y[n]$ of the system (yes/no)? If your answered yes, describe this relationship using a mathematical equation. If you answered no, briefly explain why.

(10 pts) c) Is there a relationship between your answer in a) and the 6-point DFTs $X_6[k]$ and $H_6[k]$ of the input and the unit impulse response, respectively? (Answer yes/no.) If your answered yes, describe this relationship using a mathematical equation. If you answered no, briefly explain why.

Table

CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad (1)$$

$$\text{Inverse F.T.: } x(t) = \int_{-\infty}^{\infty} \mathcal{X}(f)e^{j2\pi ft}df \quad (2)$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (3)$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \quad (4)$$

Discrete Fourier Transform

Let $x[n]$ be a periodic discrete-time signal with period N

$$\text{D.F.T.: } X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} \quad (5)$$

$$\text{Inverse D.F.T.: } x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \quad (6)$$

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (7)$$

Student-Created Cheat Sheet

This is the cheat-sheet that was created by students on Rhea. I do not guarantee that it is completely free of mistakes, but it does not contain anything that would prevent you from being able to solve the exam correctly.

Potentially Useful Formulae

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}, \ |a| < 1 \qquad \sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}, \ |a| < 1$$

$$W_N^{kn} = e^{-j\frac{2\pi}{N}kn}$$

Euler's Formula $$e^{j\omega} = cos(\omega) + jsin(\omega)$$

$$cos(\omega) = \frac{e^{j\omega}+e^{-j\omega}}{2} \qquad sin(\omega) = \frac{e^{j\omega}-e^{-j\omega}}{2j}$$

$$\mathcal{F}[\frac{rect(t-\frac{T}{2})}{T}] \Rightarrow Tsinc(Tf)e^{-j2\pi f\frac{T}{2}}$$

DFT $$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$$

IDFT $$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn}$$

DTFT $$X(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$$

IDTFT $$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$$

Z-Transform $$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

Time Shift Property of Z-Transform $$x[n-n_0] => X(z)z^{-n_0}$$

Comb/Rep $$rep_T(x(t)) = \sum_{k=-\infty}^{\infty} x(t-kT)$$

Comb/Rep $$comb_T(x(t)) = \sum_{k=-\infty}^{\infty} x(kT)\delta(t-kT)$$

Comb/Rep $$rep_T(x(t)) <=> \frac{1}{T}comb_{\frac{1}{T}}(X(f))$$

Comb/Rep $$comb_T(x(t)) <=> \frac{1}{T}rep_{\frac{1}{T}}(X(f))$$

Circular Convolution $$f[n] *_N g[n] = \sum_{k=0}^{N-1} f[k]g[(n-k)mod\ N]$$

Short Time Fourier Transform $$X[k,m] = \sum_{n=-\infty}^{\infty} x[n]w[n-m]e^{-j\frac{2\pi}{N}kn}$$

CSFT $$f(x,y) <=> F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-j2\pi(ux+vy)}\ dxdy$$

ICSFT $$F(u,v) <=> f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}\ dudv$$

Sinc $$sinc(\theta) = \frac{sin(\pi\theta)}{\pi\theta}$$

-SCRATCH -
(will not be graded)

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