Ch 1

Thursday, August 23, 2007
3:05 PM

1.2 Transformation of independat variable
1.3 Exp & Sinusoidal signals

1. Signal Power and Energy

Let $t_1 < t_2$

Energy expended from t_1 to $t_2 = \int_{t_1}^{t_2} |x(t)|^2 \, dt$

(magnitude of complex num)

ex. $|j| = 1 \quad |e^{\frac{5\pi}{3}}| = 1$

Average power

Average power in time interval $t_1 < t < t_2$

$$\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

Total energy

E_∞ is defined as the total energy expended in the time $-\infty < t < \infty$

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

Total Power

P_∞ is the average power of the signal over the interval $-\infty < t < \infty$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

For DT Signals

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \qquad P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

Timeshift

$x(t) \longrightarrow y(t) = x(t - t_0)$

"time delay" of t_0, $t_0 \in \mathbb{R}$

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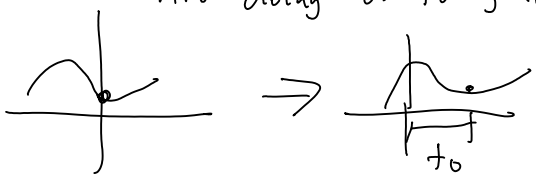


in DT:

$$x[n] \rightarrow y[n] = x[n - n_0]$$

"time delay" by n_0 · a positive or negative integer

Ex $y(t) \begin{cases} \sin(2\pi 440 t) & 0 \le t < 5 \\ 0 & \text{, else} \end{cases}$

Time Reversal

$$x(t) \rightarrow y(t) = x(-t)$$
$$x[n] \rightarrow y(n) = x(-n)$$

$$x(t) = \begin{cases} \sin(2\pi 440 t) & \text{, } 0 \le t < 1 \\ \sin(2\pi 880 t) & \text{, } 1 \le t < 2 \\ \sin(2\pi 1760 t) & \text{, } 2 \le t < 3 \\ 0 & \text{, else} \end{cases}$$

to reverse a song, it needs to be reversed and then shifted so that the song starts at zero

$z(t) = x(\ ?\ )$ step by step

$y(t) = x(-t)$ — reversal $\quad y(\square) = x(-\square)$

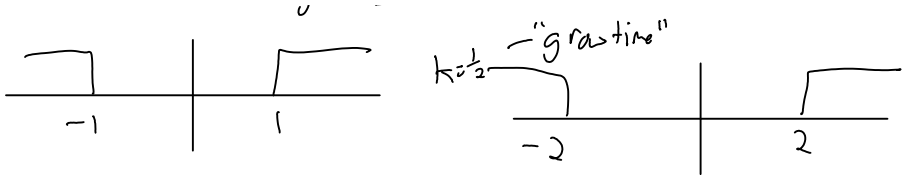
Combine $z(t) = y(t-3)$ - shift

$z(\square) = y(\square - 3) = x(-(t-3)) = x(-t+3)$

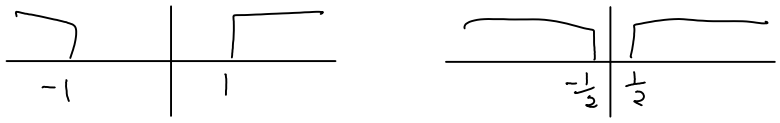
Time Scaling

$$x(t) \rightarrow y(t) = x(kt)$$
$$x[n] \rightarrow y[n] = x[kn]$$

— | ⌐— , ↓ — "granting"



k=2



Generalized $x(t) \rightarrow y(t) = x(at+b)$

stretched if $|a| < 1$

Compressed if $|a| > 1$

reversed if $a < 0$

shifted if $b \neq 0$

Exercise

System 1

$x(t) \rightarrow$ [ ] $\rightarrow y(t) = x(2t)$

System 2

$x(t) \rightarrow$ [ ] $\rightarrow y(t) = x(t-7)$

$x(t) \rightarrow$ [S1] $\xrightarrow{y(t) = x(2t)}$ [S2] $\rightarrow z(t) = y(t-7)$

$= x(2(t-7))$

$x(t) \rightarrow$ [S2] $\xrightarrow{y(t) = x(t-7)}$ [S1] $\rightarrow z(t) = y(2t)$

$y(\square) = x(\square - 7)$

$= x(2t-7)$

Even / odd signal

$x(t)$ or $x[n]$ is even if $x(-t) = x(t)$

$x[-n] = x[n]$

odd if $x(-t) = -x(t)$

$x[-n] = -x[n]$

ex. $x(t) = \frac{t^2(1+t^4)}{t \sin t}$ even or odd

$$x(-t) = \frac{(-t)^2(1+(-t)^4)}{-t\sin(-t)} = \frac{t^2(1+t^4)}{-t \cdot -\sin(t)} =$$

$$\frac{t^2(1+t^4)}{t\sin t}$$ so even

Fact Every signal can be written as the sum of an even and an odd signal

$$x(t) = \underbrace{x_1(t)}_{\text{even}} + \underbrace{x_2(t)}_{\text{odd}}$$

Proof

$$x(t) = \underbrace{\frac{x(t)+x(-t)}{2}}_{\text{even}} + \underbrace{\frac{x(t)-x(-t)}{2}}_{\text{odd}}$$

Periodic Signal

A CT signal is called periodic if there exists a $T>0$ ("period") such that $x(t+T) = x(t)$ for all values of t

The "fundamental period" is the smallest period of all periods of a signal (Denote by T_0)

Similarly, for DT signals

Fact! $x[n] = \cos n$ is not periodic