ECE 440L

Experiment X: Binary Phase Shift Keying and Frequency Spreading

I. OBJECTIVES

1. Construct and test the BPSK system described in Figures 1 and 2.
   a. Prove that the audio is being sampled at approximately 1 kHz.
   b. Use proper cut-off frequencies for anti-aliasing and reconstruction filters.
   c. Adjust system to obtain signal voltages with the appropriate amplitude and phase.

2. Obtain time and frequency domain displays for the 25 kHz and 50 kHz carriers.

3. Using 4-bit linear PCM and a 100 Hz squarewave for the analog input signal, obtain time and frequency domain (PSD) displays for:
   a. The PCM Encoder (TTL) data and the PCM (NRZ) data.
   b. The BPSK Channel.
   c. The output of the demodulating multiplier and the PCM reconstruction filter.
   d. The output of the PCM Decoder and reconstruction filter.

4. Apply an audio input. Connect system output to the scope and audio amplifier.
   a. Adjust system gains to obtain signals of the appropriate amplitude and phase.
   b. Obtain time domain and PSD displays for the BPSK Channel.
   c. Adjust the output filter to, alternately, hear and suppress the distortion.

5. Using the audio input, compare the BPSK Channel PSD and sound “quality” for:
   a. 25 kHz carrier frequency vs. 50 kHz carrier frequency)
   b. 7-bit linear vs. 4 bit compander.
6. Construct and test the BPSK-Spread-Spectrum system described in Figures 1 and 2.
   a. Use proper cut-off frequencies for anti-aliasing and reconstruction filters.
   b. Adjust system to obtain signal voltages with the appropriate amplitude and phase.

7. Using 4-bit linear PCM and a 100 Hz squarewave for the analog input signal, obtain time and frequency domain (PSD) displays for:
   a. The BPSK input to the spreader..
   b. The spread-spectrum signal in the BPSK-Spread Spectrum channel.

II. INTRODUCTION

Figures 1 and 2 show the block diagram and connection diagram of the BPSK system to be investigated for this experiment. The purpose of the system is to digitize audio range signals and transmit the PCM data stream via BPSK. The carrier frequency shown is 50 kHz and the the bit-rate is approximately 1000 bits/second.

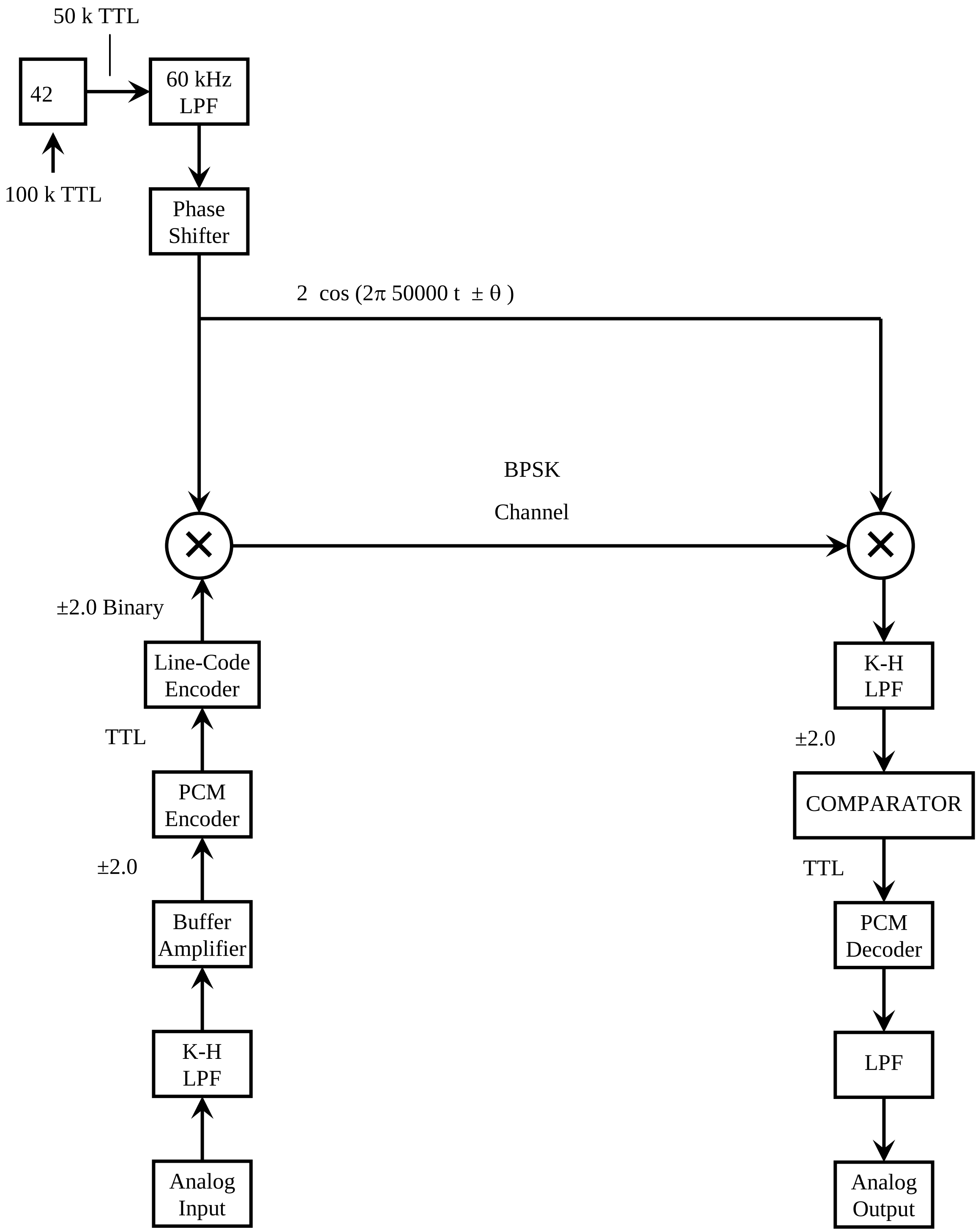


Fig.1. Block Diagram for BPSK System.

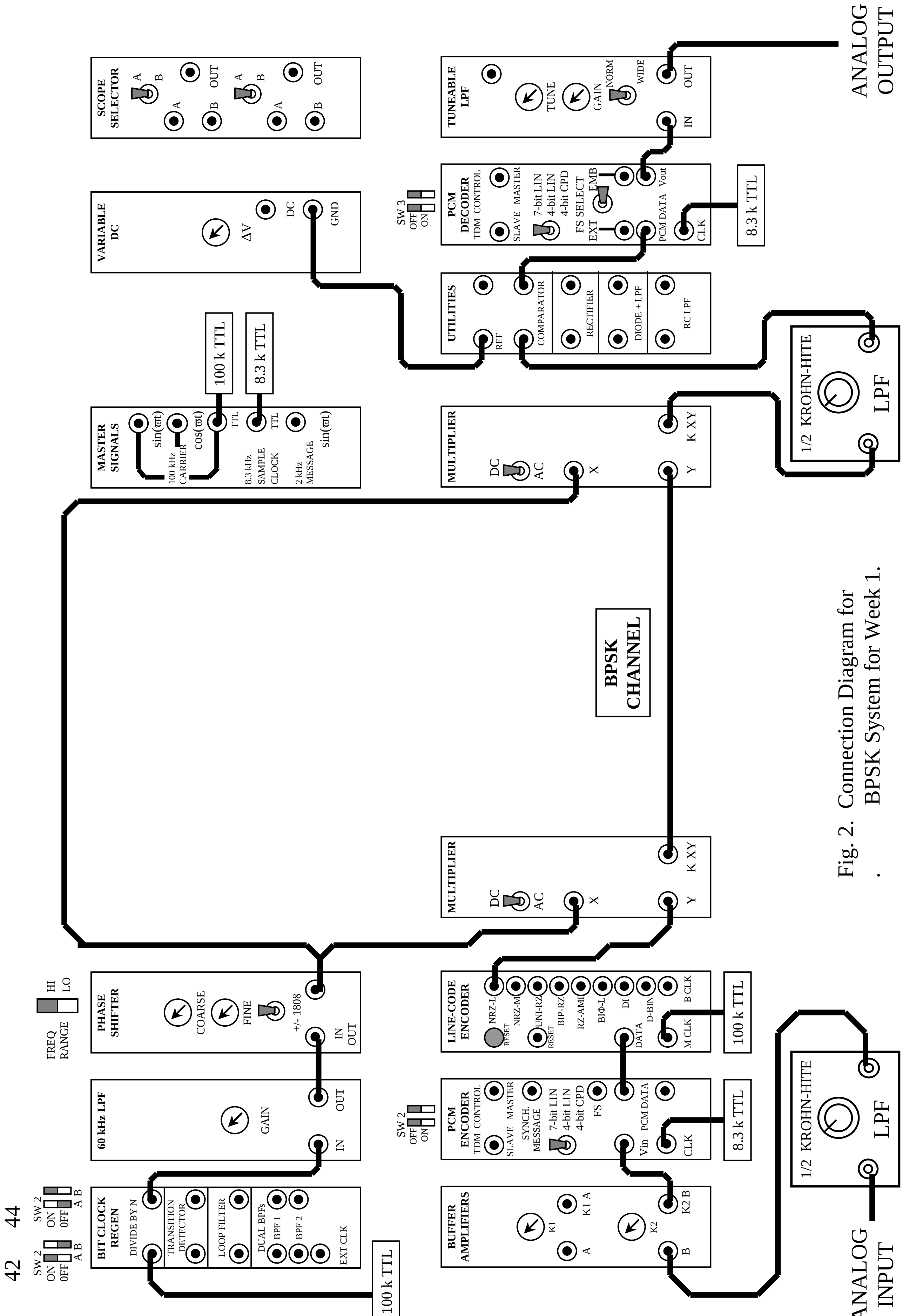


Fig. 2. Connection Diagram for BPSK System for Week 1.

Figures 3 and 4 show the block diagram and connection diagram of the Frequency Spread BPSK system to be investigated for this experiment. The purpose of the system is to digitize audio range signals and transmit the PCM data stream via Spread Spectrum BPSK. The carrier frequency shown is 50 kHz and the the bit-rate is approximately 1000 bits/second.

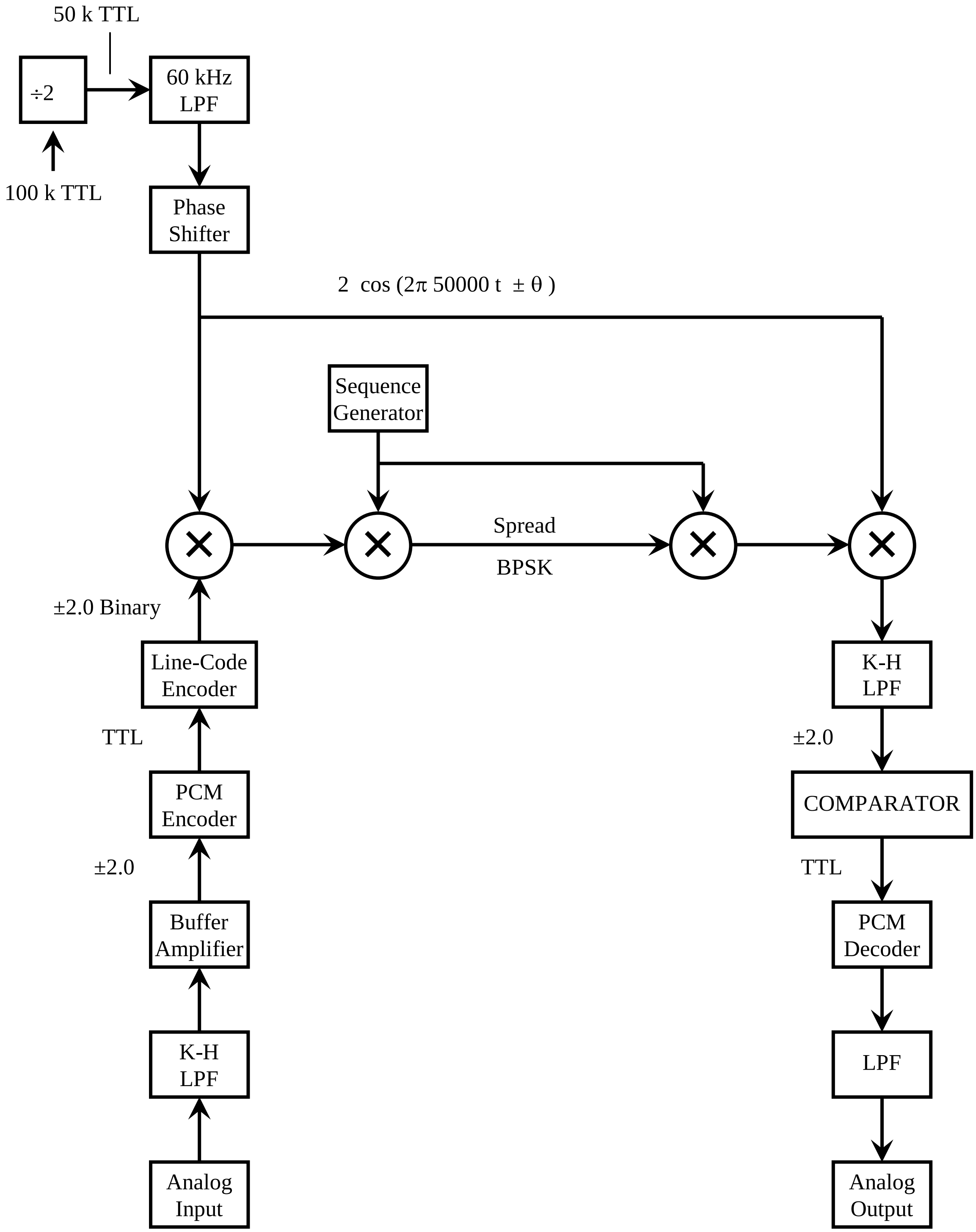


Fig.3. Block Diagram for Spread BPSK System.

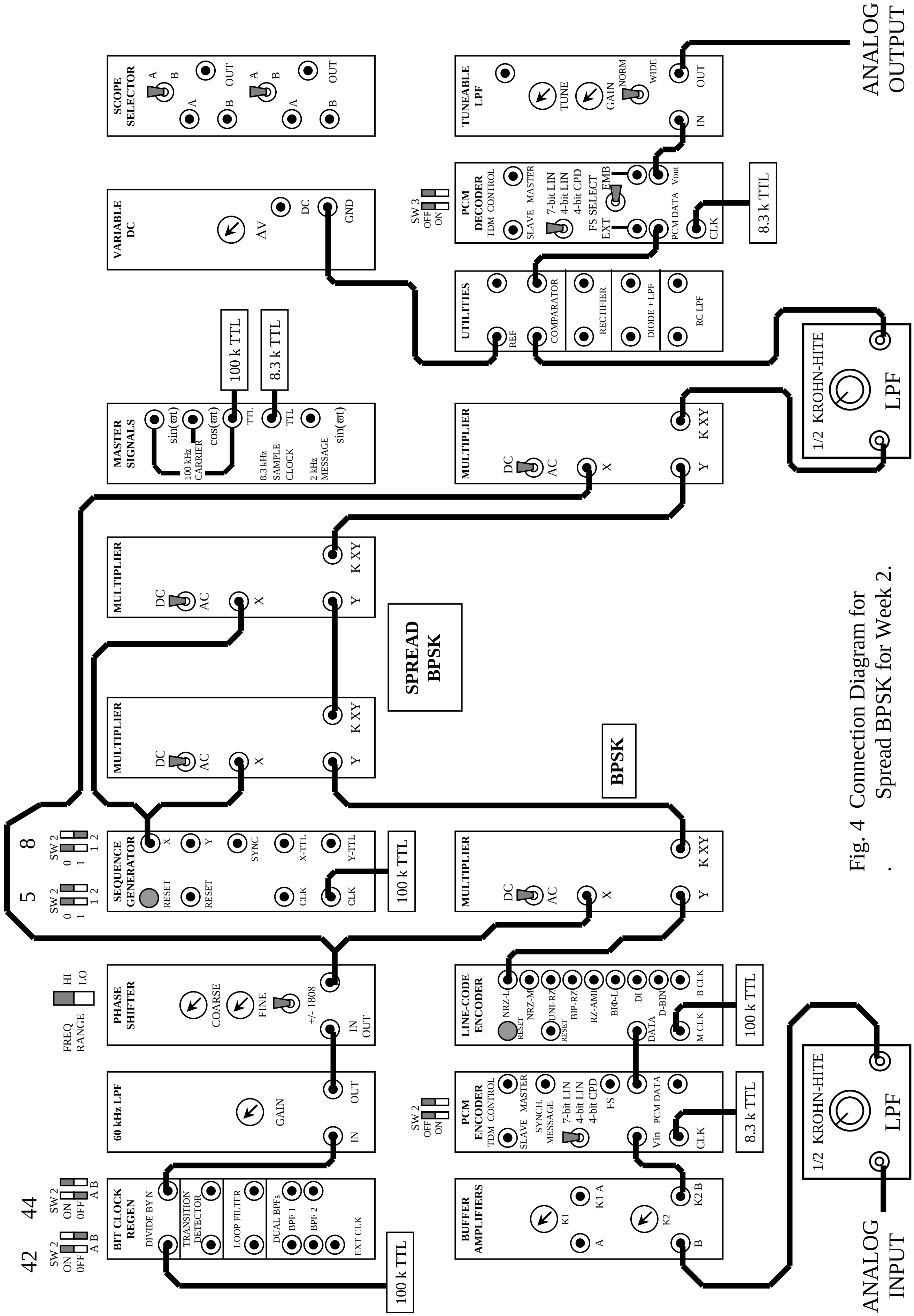


Fig. 4 Connection Diagram for Spread BPSK for Week 2.

III. PRELAB (See Appendix)

1. Write the theoretical Power Spectral Density for a BPSK signal when the shift angle is 180 degrees. State it or derive it. Provide clear definitions of the variables and a reference for your starting point.

   HINT: It looks like [sin(x)/x $]^2$.

2. In Figure 2, a 50 kHz squarewave is input to the 60 kHz low pass filter. The filter is effective at rejecting frequencies higher than 60 kHz.

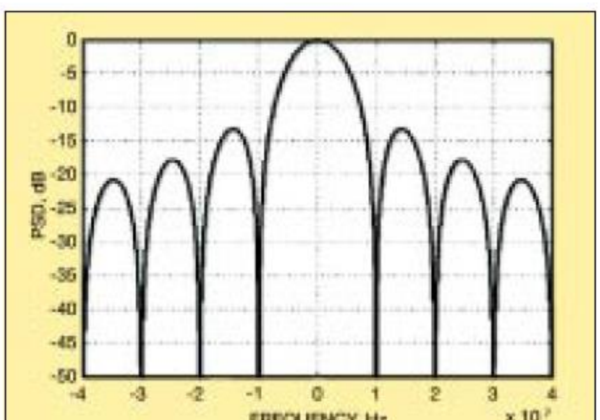
   a. Does it make sense that the output of the filter is a 50 kHz sinewave? Prove your answer.

   b. A 25 kHz squarewave input to the filter also produces a clean 25 kHz sinewave. How is that possible?

3. Write the theoretical Power Spectral Density for a BPSK signal with shift angle of 180 degrees, carrier of 25 kHz and bit rate of about 1 kHz that has been spread by a PN sequence with a bit rate of about 100 kHz. State it or derive it. Provide clear definitions of the variables and a reference for your starting point.

IV. EXPERIMENT.

Perform experiments required to meet the stated objectives.
Record your procedures and results.



Unfiltered BPSK Spectrum.
From RF Design, April 2004

APPENDIX

Power spectral density for a BPSK signal

We consider an asynchronous binary signal (ABS) process with each pulse having width T_b. Let the random variable X_n denote the height of the n^{th} pulse taking values ± 1 with equal probability. We call the process asynchronous because the displacement D of the 0^{th} pulse is $U[0, T_b]$. The ABS process can be mathematically written as:

$$X(t) = \sum_n X_n \Pi\left(\frac{t - D - nT_b}{T_b}\right) \tag{1}$$

where the rectangular window $\Pi(t)$ is defined as

$$\Pi(t) = \begin{cases} 1 & |t| \leq 1/2 \\ 0 & \text{otherwise.} \end{cases}$$

In the definition of ABS process we assume that the levels of different pulses are independent of each other and they are in turn independent of the random displacement D. Note that ABS is a Wide Sense Stationary process with mean $E[X(t)] = 0$ and

$$R_{XX}(t_1, t_2) = \begin{cases} 1 - \frac{|t_1 - t_2|}{T_b} & |t_1 - t_2| \leq T_b \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

With ABS as our random binary signal, the BPSK signal can be given by:

$$S(t) = \sqrt{(2)} X(t) \cos(2\pi f_c t + \Theta) \tag{3}$$

where Θ is $U[0, 2\pi]$, again taking value independent of random variables $X_i's$ and D. Under this set up of the BPSK signal it can be shown that the signal $S(t)$ is a WSS process. Remember that a process is WSS if the mean is independent of time and autocorrelation function depnds only on the time difference $|t_1 - t_2|$. Once we have shown that the process is WSS, we can apply Wiener-Khinchine Theorem to calculate the power spectral density of $S(t)(\mathcal{F}(R_{SS}(t_1, t_2)) = S_{SS}(f))$. Infact it can be shown that the power spectral density of BPSK signal is given by:

$$S_{SS}(f) = \frac{1}{2} T_b \text{sinc}^2((f - f_c)T_b) + \frac{1}{2} T_b \text{sinc}^2((f + f_c)T_b) \tag{4}$$

Power spectral density for a Spread Spectrum BPSK signal

We *spread* the BPSK signal by multiplying the signal with a random ABS signal $C(t)$ having pulse width $T_c << T_b$. We can represent the spread BPSK signal as:

$$S(t) = \sqrt{(}2)X(t)C(t)\cos(2\pi f_c t + \Theta) \tag{5}$$

We further assume that the evolution of random process $X(t)$ is independent of the random process $C(t)$(if both the processes are derived from a same common clock, then the independence assumption is not strictly valid). Under these assumptions, the autocorrelation function for $S(t)$ is given by

$$R_{SS}(t_1, t_2) = R_{XX}(t_1, t_2)R_{CC}(t_1, t_2)\cos(2\pi f_c|t_1 - t_2|) \tag{6}$$

or equivalently (since $X(t)$ and $C(t)$ are WSS),

$$R_{SS}(\tau) = R_{XX}(\tau)R_{CC}(\tau)\cos(2\pi f_c\tau) \tag{7}$$

But from the above equation, we conclude that $S(t)$ is again a WSS process, and therefore the power spectral density of $S(t)$ is given by

$$S_{SS}(f) = S_{XX}(f) * S_{CC}(f) * \mathcal{F}(\cos(2\pi f_c\tau)) \tag{8}$$

where $*$ denotes the convolution operation. Since we assumed that $T_c << T_b$, the spread/spectral width of $S_{CC}(f)$ would be much higher than that of $S_{XX}(f)$. Therefore we can approximate their convolution by simply $S_{CC}(f)$. Thus the spectrum of spread BPSK can be approximately given as:

$$S_{SS}(f) = \frac{1}{2}T_c\mathrm{sinc}^2((f - f_c)T_c) + \frac{1}{2}T_c\mathrm{sinc}^2((f + f_c)T_c) \tag{9}$$

REFERENCES

[1] R.E. Ziemer and W.H. Tranter, Principles of Communications, 5^{th} edition, Pages 260-261, 432-436.