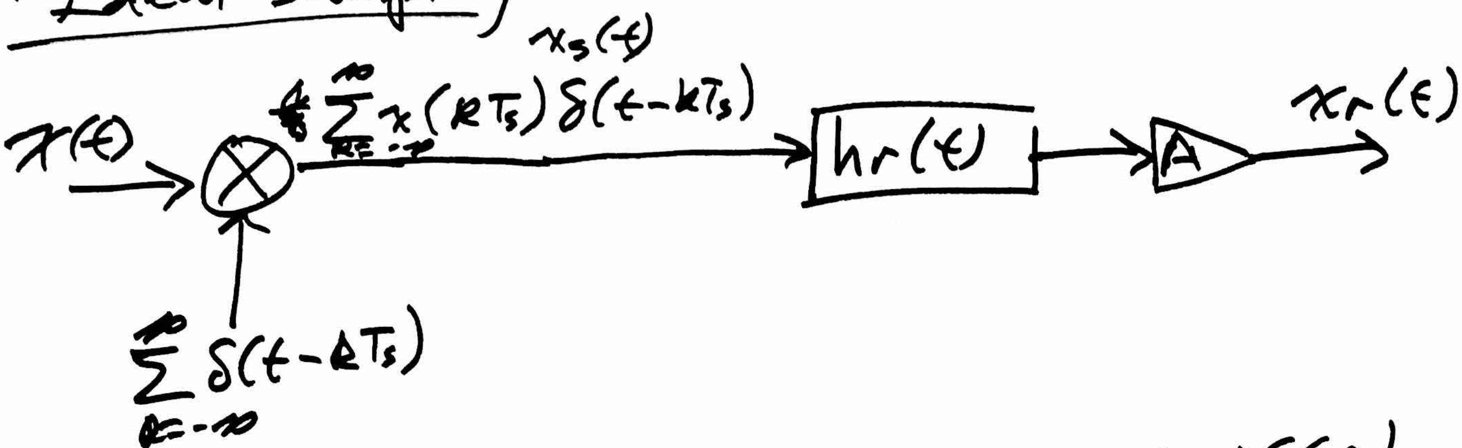
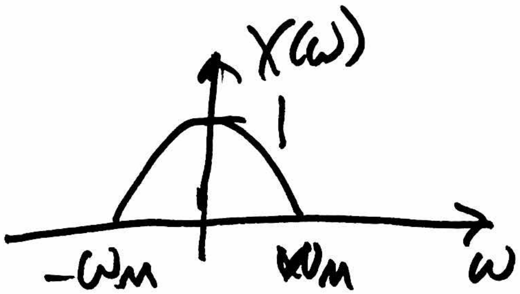
July 21, 2016

Sampling and Reconstruction

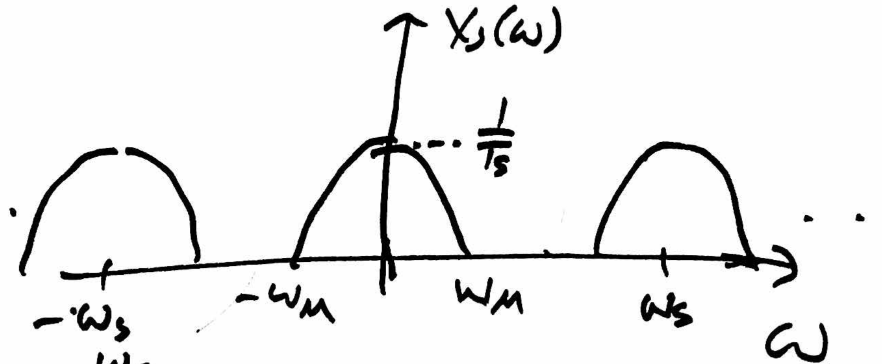
→ Ideal sampling



If $x(t)$ is bandlimited such that $X(\omega)=0$ for $|\omega|>\omega_M$ and $\omega_s > 2\omega_M$, then $x_r(t) = x(t)$.



and



Choose $H_r(\omega) = \begin{cases} 1, & |\omega| < \frac{\omega_s}{2} \\ 0, & \text{else} \end{cases}$

$A = T_s$, then $x_r(t) = x(t)$

$$h_r(t) = \mathcal{F}^{-1}\{H_r(\omega)\} = \frac{\sin \frac{\omega_s}{2} t}{\pi t}$$

$$\omega_s = \frac{2\pi}{T_s}$$

$$\frac{\omega_s}{2} = \frac{\pi}{T_s}$$

$$= \frac{\sin\left(\frac{\pi}{T_s} t\right)}{\pi t}$$

$$x_r(t) = \left(x_s(t) * h_r(t)\right) \cdot A$$

$$= \left(\left(\sum_{k=-\infty}^{\infty} x(kT_s) \cdot \delta(t-kT_s)\right) * \frac{\sin \frac{\pi}{T_s} t}{\pi t}\right) T_s$$

$$= T_s \sum_{k=-\infty}^{\infty} x(kT_s) \left(\frac{\sin \frac{\pi}{T_s} t}{\pi t}\right) * \delta(t-kT_s)$$

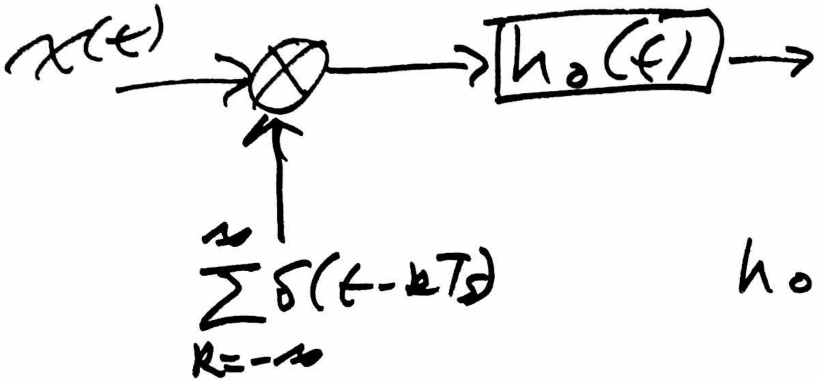
$$= T_s \sum_{k=-\infty}^{\infty} x(kT_s) \frac{\sin\left(\frac{\pi}{T_s}(t-kT_s)\right)}{\pi(t-kT_s)}$$

$$= \sum_{k=-\infty}^{\infty} x(kT_s) \frac{\sin\left(\frac{\pi}{T_s}(t-kT_s)\right)}{\frac{\pi}{T_s}(t-kT_s)}$$

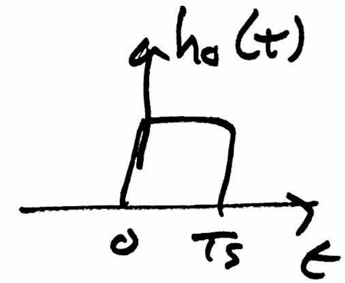
$$= x(t)$$

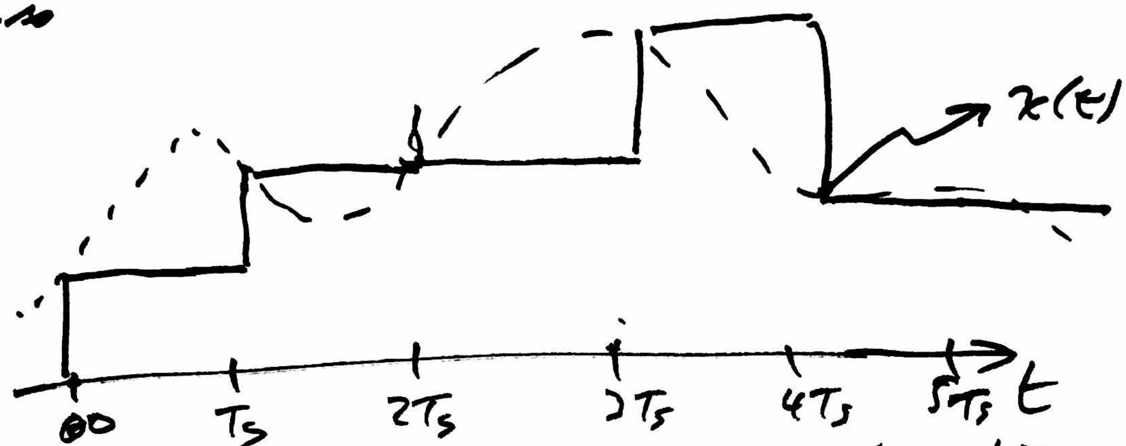
Sampling Theorem

Non-ideal reconstruction



$$h_0(t) = \begin{cases} 1, & 0 < t < T_s \\ 0, & \text{else} \end{cases}$$

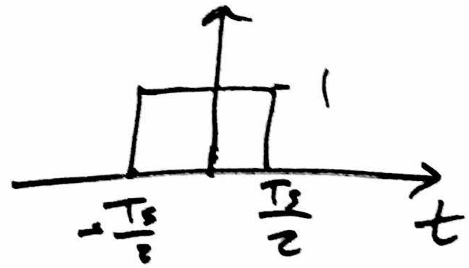




Zero-order hold reconstruction
very easy to implement, but we get a rough reconstruction.

• Zero-order-hold ~~effects~~ effects on frequency domain

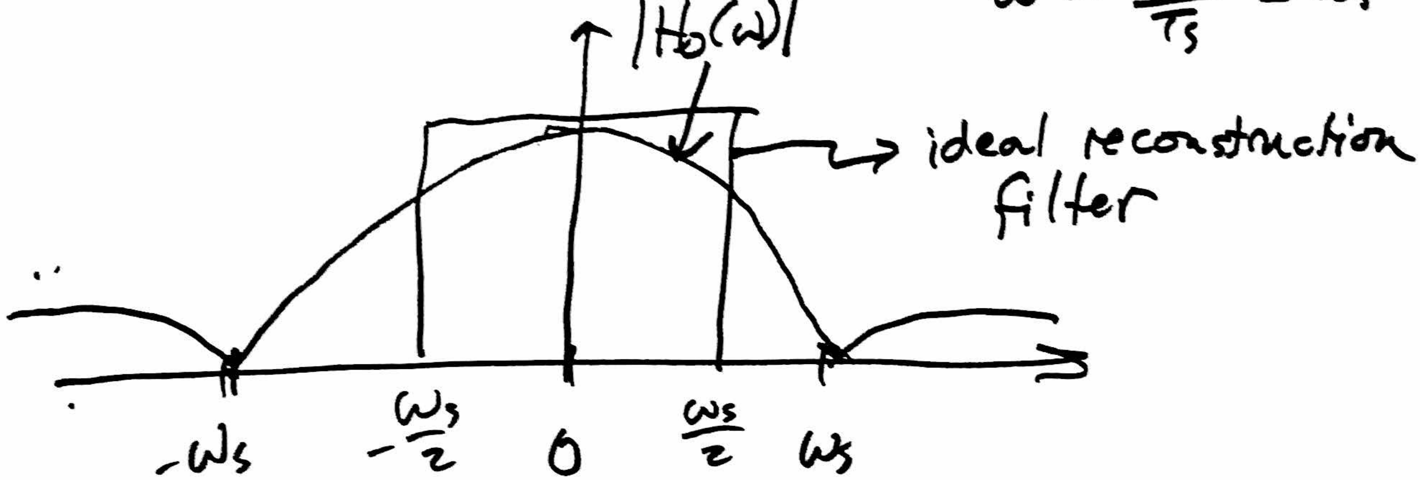
$h_0\left(t+\frac{T_s}{2}\right)$



$$H_0(\omega) = e^{j\omega\frac{T_s}{2}} \cdot \frac{2\sin\left(\omega\frac{T_s}{2}\right)}{\omega}$$

$$\omega\frac{T_s}{2} = \pi$$

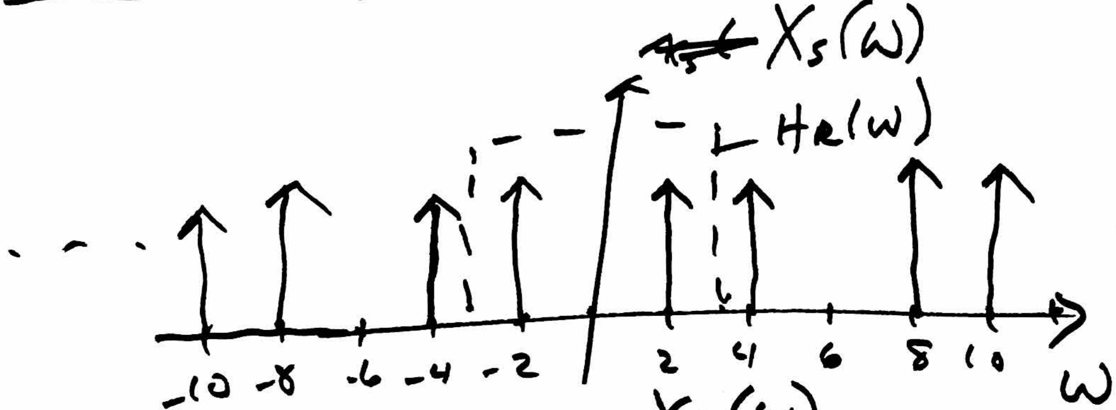
$$\omega = \frac{2\pi}{T_s} = \omega_s$$



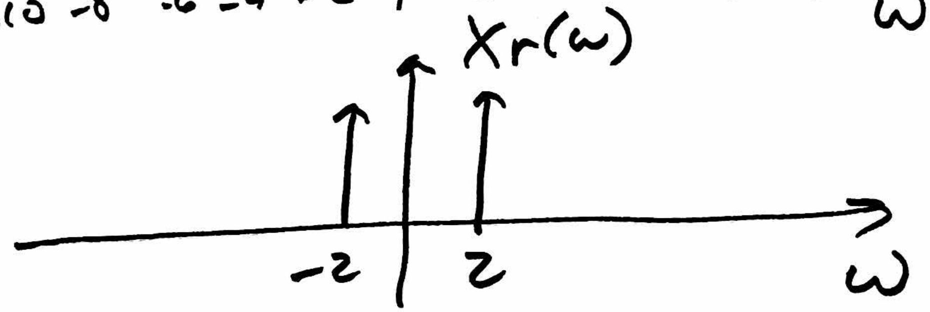
Aliasing

→ given that $H_R(\omega) = H_r(\omega) = \begin{cases} 1, & |\omega| < \frac{\omega_s}{2} \\ 0, & \text{else} \end{cases}$

Ex $x(t) = \cos(4t)$ $\quad \omega_s = 6 \quad \omega_N = 8$ ← Nyquist rate



$H_R(\omega) = H_r(\omega) = \begin{cases} 1, & |\omega| < 3 \\ 0, & \text{else} \end{cases}$



$x_r(t) = \cos(2t)$

Choosing $\omega_s = 6$ (below Nyquist) misrepresents the original signal.

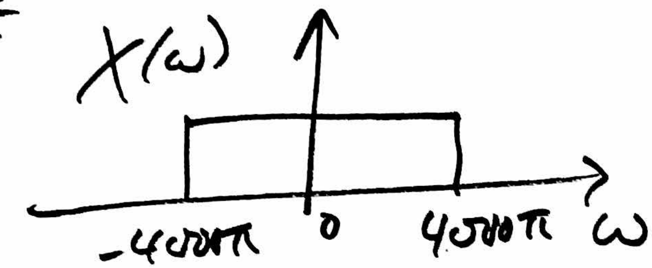
OW 7.3 — Find ω_N ~ Nyquist rate

a) $x(t) = 1 + \cos(2000\pi t) + \sin(4000\pi t)$

$\sin(4000\pi t)$ is the highest frequency term,

$$\Rightarrow \omega_N = 2 \cdot 4000\pi = 8000\pi$$

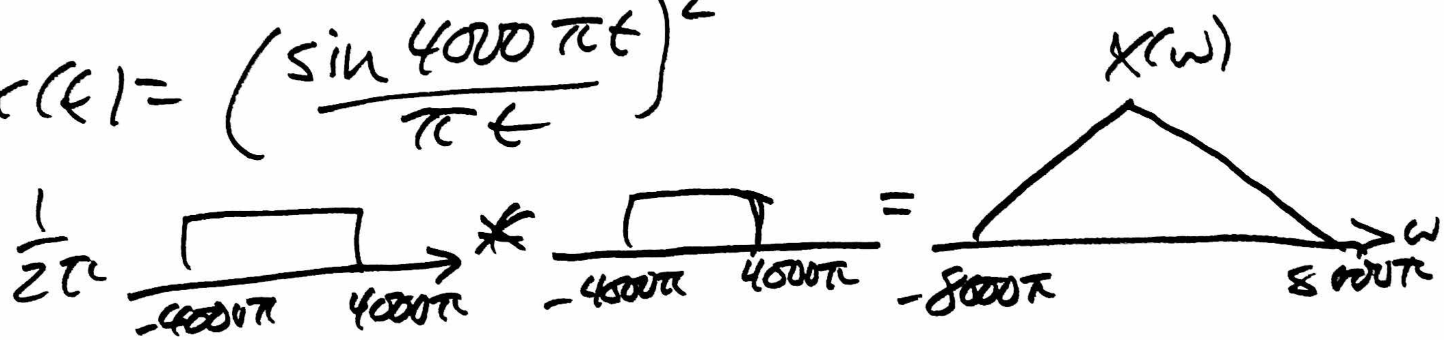
b) $x(t) = \dfrac{\sin 4000\pi t}{\pi t}$



$$\Rightarrow \omega_N = 2 \cdot 4000\pi = 8000\pi$$

$X(\omega) = 0$ for $|\omega| > 4000\pi$

c) $x(t) = \left(\dfrac{\sin 4000\pi t}{\pi t}\right)^2$



$$\omega_N = 2 \cdot 8000\pi = 16000\pi$$