Midterm Examination 1
ECE 301
Division 2, Fall 2006
Instructor: Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 50 minutes to complete the 6 questions contained in this exam, for a total of up to 105 points. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.

3. This booklet contains 11 pages. The last three pages contain a table of formulas and properties. You may tear out these three pages **once the exam begins**.

4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden.

Name:________________________________

Email:________________________________

Signature:____________________________

(15 pts) **1.** The output y(t) of a system is related to the input x(t) by the equation

$$y(t) = x\left(\frac{t}{3}\right).$$

a) Is the above system time invariant? (Answer yes/no and prove it.)

b) The above system is (check all that apply, no justification needed.)

linear	
stable	
invertible	
with memory	
causal	
LTI	

(15 pts) **2.** The unit impulse response of an LTI system is $h(t) = u(t)$. What is the output of the system when the input is $x(t) = e^{-2t}u(t)$? (Justify your answer.)

(20 pts) **3.** Consider an input $x[n]$ and a unit impulse response $h[n]$ given by

$$\begin{aligned} x[n] &= \left(\frac{1}{2}\right)^{n-2} u[n-2], \\ h[n] &= u[n+2]. \end{aligned}$$

Determine the output $y[n] = x[n] * h[n]$. (You may leave your answer in unsimplified fraction form.)

(20 pts) **4.** Indicate whether each of the following signals is periodic or not. (No justification needed.) If they are periodic, write down what their fundamental period is. (Again, no justification needed.)

	Periodic yes/no?	Fundamental Period
$x[n] = e^{6\pi n+1}$		
$x[n] = \sin(\frac{6\pi}{8}n + 1)$		
$x[n] = e^{j(\frac{6}{8}n+1)}$		
$x(t) = e^{-t}$		
$x(t) = e^{j(\pi t-1)}$		
$x(t) = \sum_{k=-\infty}^{\infty} e^{-(2t-k)}u(2t-k)$		

(10 pts) **5.** Find the Fourier series coefficients a_k of the DT signal

$$x[n] = 1 + 7\cos(\pi n).$$

(25 pts) **6.** Consider a continuous-time LTI system whose frequency response is

$$H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt = \frac{\sin(4\omega)}{\omega}.$$

The input to this system is a periodic signal

$$x(t) = \begin{cases} 1, & 0 \leq t < 4 \\ -1, & 4 \leq t < 8 \end{cases}$$

with period $T = 8$. Determine the Fourier series coefficients of $x(t)$ and determine the corresponding system output $y(t)$. (Use the next page if you need extra space.)

1 CT Signal Energy and Power

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \tag{1}$$

$$P_\infty = \frac{1}{2T} \lim_{T \to \infty} \int_{-T}^{T} |x(t)|^2 \, dt \tag{2}$$

2 Fourier Series of CT Periodic Signals with period T

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t} \tag{3}$$

$$a_k = \frac{1}{T} \int_0^T x(t) e^{-jk\left(\frac{2\pi}{T}\right)t} dt \tag{4}$$

3 DT Signal Energy and Power

$$E_\infty = \sum_{n=-\infty}^{\infty} |x[n]|^2 \tag{5}$$

$$P_\infty = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \tag{6}$$

4 Fourier Series of DT Periodic Signals with period N

$$x[n] = \sum_{k=0}^{N-1} a_k e^{jk\left(\frac{2\pi}{N}\right)n} \tag{7}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\left(\frac{2\pi}{N}\right)n} \tag{8}$$

5 Properties of LTI systems

- LTI systems commute.

- The response of an LTI system with unit impulse response h to a signal x is the same as the response of an LTI system with unit impulse response x to the signal h.

- An LTI system consisting of a cascade of k LTI systems with unit impulse responses $h_1, h_2, \ldots, h_k$ respectively, is the same as an LTI system with unit impulse response $h_1 * h_2 * \ldots * h_k$.

- The response of a CT LTI system with unit impulse response $h(t)$ to the signal e^{st} is $H(s)e^{st}$ where $H(s) = \int_{-\infty}^{\infty} h(\tau)e^{-s\tau}d\tau$.

- The response of a DT LTI system with unit impulse response $h[n]$ to the signal z^n is $H(z)z^n$ where $H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$.