(22 pts) **1.** Let $x(t)$ and $y(t)$ be the input and the output of a continuous-time system, respectively. Answer each of the questions below with either yes or no (no justification needed).

	Yes	No	
If $y(t) = x(2t)$, is the system causal?	[ ]	[x]	
If $y(t) = (t+2)x(t)$, is the system causal?	[x]	[ ]	
If $y(t) = x(-t^2)$, is the system causal?	[x]	[ ]	X $y(-\frac{1}{2}) = x(-\frac{1}{4})$
If $y(t) = x(t) + t - 1$, is the system memoryless?	[x]	[ ]	
If $y(t) = x(t^2)$, is the system memoryless?	[ ]	[x]	
If $y(t) = x(t/3)$, is the system stable?	[x]	[ ]	
If $y(t) = tx(t/3)$, is the system stable?	[x]	[ ]	X
If $y(t) = \int_{-\infty}^{t} x(\tau)d\tau$, is the system stable?	[ ]	[x]	
If $y(t) = \sin(x(t))$, is the system time invariant?	[x]	[ ]	
If $y(t) = u(t) * x(t)$, is the system LTI?	[ ]	[x]	X Always true!
If $y(t) = (tu(t)) * x(t)$, is the system linear?	[ ]	[x]	X True, because convolution is linear

12

(15 pts) **2.** An LTI system has unit impulse response $h(t) = u(t+2)$. Compute the system's response to the input $x(t) = e^{-t}u(t)$. (Simplify your answer until all $\sum$ signs disappear.)

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} e^{-\tau} u(\tau)\, u(t-\tau+2)\, d\tau$$

$$= \int_{0}^{\infty} e^{-\tau}\, u(t-\tau+2)\, d\tau$$

* When $t-\tau+2>0$, $u(t-\tau+2)=1$, else $u(t-\tau+2)=0$

* so $\tau < t+2$, for $u(t-\tau+2)=1$

$$= \left(\int_{0}^{t+2} e^{-\tau}\, d\tau\right) u(t) \quad u(t+2)$$

$$= \left(-e^{-\tau}\Big|_{0}^{t+2}\right) u(t)$$

$$= \left(-e^{-(t+2)} + 1\right) u(t)$$

$$= \left(1 - e^{-(t+2)}\right) u(t)$$

15

(15 pts) **3.** Compute the energy and the power of the signal $x(t) = \frac{3e^{jt}}{1+j}$.

$$E_\infty = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt$$

$$|x(t)| = \frac{|3e^{jt}|}{|1+j|}$$

$$= \frac{3}{\sqrt{2}}$$

$$E_\infty = \int_{-\infty}^{\infty} \left(\frac{3}{\sqrt{2}}\right)^2 dt$$

$$= \int_{-\infty}^{\infty} \frac{9}{2} \, dt$$

$$= \frac{9}{2}t \Big]_{-\infty}^{\infty}$$

$$E_\infty = \infty$$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \left(\frac{3}{\sqrt{2}}\right)^2 dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} \frac{9}{2} \, dt$$

$$= \lim_{T \to \infty} \frac{1}{2T} \left(\frac{9}{2}t \Big|_{-T}^{T}\right.$$

$$\frac{9}{2} \left(\lim_{T \to \infty} \frac{2T}{2T} \right.$$

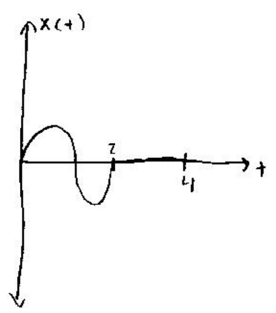
$$P_\infty = \frac{9}{2}$$

2

(15 pts) **4.** Compute the coefficients a_k of the Fourier series of the signal $x(t)$ periodic with period $T = 4$ defined by

$$x(t) = \begin{cases} \sin(\pi t), & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}.$$

(Simplify your answer as much as possible.)



$$\sin(\pi t) = \frac{e^{j\pi t} - e^{-j\pi t}}{2j}$$

$$= \frac{1}{2j} e^{j\pi t} - \frac{1}{2j} e^{-j\pi t}$$

$a_k = 0$ between $2 < t \le 4$

Therefore,

$a_1 = -\frac{2}{j}$ $\quad a_{-1} = \frac{2}{j}$

Since the average of $x(t)$ over one period is zero

$a_0 = 0$

$$a_k = \frac{1}{4}\int_0^4 x(t)\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^2 \sin(\pi t)\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{4}\int_0^2 \frac{e^{j\pi t} - e^{-j\pi t}}{2j}\, e^{-jk\frac{\pi}{2}t}\, dt$$

$$= \frac{1}{8j}\int_0^2 e^{j\pi(1-\frac{k}{2})t} - e^{-j\pi(1+\frac{k}{2})t}\, dt$$

If $k \neq 2, -2$

$$= \frac{1}{8j}\left[\frac{e^{j\pi(1-\frac{k}{2})t}}{j\pi(1-\frac{k}{2})} - \frac{e^{-j\pi(1+\frac{k}{2})t}}{j\pi(1+\frac{k}{2})}\right]_0^2$$

$$= \frac{-1}{8\pi}\left[\frac{e^{2j\pi(1-k/2)} - 1}{(1-\frac{k}{2})} - \frac{e^{-2j\pi(1+\frac{k}{2})} - 1}{1+\frac{k}{2}}\right]$$

$$= \frac{-1}{4\pi}\left[\frac{e^{2j\pi}e^{-jk\pi} - 1}{2-k} - \frac{e^{-2j\pi}e^{-jk\pi} - 1}{2+k}\right]$$

$$= \frac{-1}{4\pi}\left[\frac{(-1)^k - 1}{2-k} - \frac{(-1)^k - 1}{2+k}\right]$$

5. A discrete-time system is such that when the input is one of the signals in the left column, then the output is the corresponding signal in the right column:

$$\begin{array}{rcl} \text{input} & & \text{output} \\ x_0[n] = \delta[n] & \rightarrow & y_0[n] = \delta[n-1], \\ x_1[n] = \delta[n-1] & \rightarrow & y_1[n] = 4\delta[n-2], \\ x_2[n] = \delta[n-2] & \rightarrow & y_2[n] = 9\delta[n-3], \\ x_3[n] = \delta[n-3] & \rightarrow & y_3[n] = 16\delta[n-4], \\ & \vdots & \\ x_k[n] = \delta[n-k] & \rightarrow & y_k[n] = (k+1)^2\delta[n-(k+1)] \text{ for any integer k.} \end{array}$$

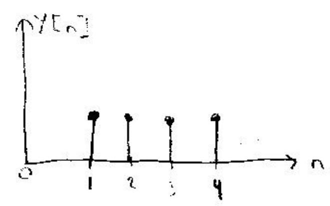
(10 pts) **a)** Can this system be time-invariant? Explain.

be precise.

No, this system is not time invariant because if a system is time invariant then the output to a shifted input $x(t-t_0)$ should $y(t-t_0)$. But if an input of $\delta[n-1]$ is sent through the system then an output of $4\delta[n-2]$ is received. which means a different time delay was the output

5

(10 pts) **b)** Assuming that this system is linear, what input $x[n]$ would yield the output $y[n] = u[n-1]$?



$$y[n] = \sum_{k=1}^{\infty} \delta[n-k]$$

let $r = k-1$

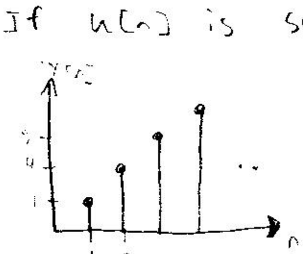
$$= \sum_{r=0}^{\infty} \delta[n-(r+1)]$$

$$= \sum_{k=0}^{\infty} \frac{y_k[n]}{(k+1)^2}$$

$$= \sum_{k=0}^{\infty} \frac{x_k[n]}{(k+1)^2}$$

0

If $u[n]$ is sent through system output will look like graph below



$$\boxed{X[n] = u[n] - \frac{1}{(n+1)^2} u[n]}$$, because system is linear