1. Let \( \pi + \mathbb{Q} = \{ \pi + q : q \in \mathbb{Q} \} \). Let \( \mathcal{A} \) be the \( \sigma \)-algebra generated by the sets \( \{ x \} : x \in \pi + \mathbb{Q} \). Without justification find all possible values for the Lebesgue measure, \( |A| \) if \( A \in \mathcal{A} \).

2. Let \( f : \mathbb{R}^n \to \mathbb{R}^n \) be continuous, and let \( G \) be an open set in \( \mathbb{R}^n \). Proof or counter-example: \( f(G) = \{ y | \exists x \in G \ with \ f(x) = y \} \) is measurable.

3. Let \( f_n \) be a sequence of functions in \( L^p, 1 \leq p < \infty \), which converge almost everywhere to a function \( f \) in \( L^p \). Show that \( f_n \to f \) in \( L^p \) iff \( ||f_n||_p \to ||f||_p \).

4. Let \( f \) be a nonnegative Lebesgue measurable function on \((0, \infty)\) such that \( f^2 \) is integrable. Let \( F(x) = \int_0^x f(t)dt \) where \( x > 0 \). Show that

\[
\lim_{x \to 0^+} \frac{F(x)}{\sqrt{x}} = 0.
\]

5. Suppose that \( \{f_n\}_{n \geq 1} \) is an equicontinuous family of functions on a compact set \( K \) such that the family of functions is point-wise bounded.

(a) Show that \( \{f_n\}_{n \geq 1} \) is uniformly bounded on \( K \).

(b) Show that \( f(x) = \inf \{ f_n(x) : n \geq 1 \} \) is uniformly continuous on \( K \).

6. Suppose \( f \) is an integrable function. Prove that

\[
\lim_{n \to \infty} \int_{-\infty}^{\infty} f(x) \cos(nx)dx = 0.
\]