Midterm Examination 1
ECE 438
Fall 2011
Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 50 minutes to complete the 4 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This exam contains 7 pages. Page 6 contains a table of formulas. Page 7 is scratch paper. You may tear out the table and the scratch paper **once the exam begins**.
4. This is a closed book exam. The use of calculators is prohibited. Cell phones, pagers, and all other electronic communication device are strictly forbidden. Ipods and PDAs are not allowed either.

Name:____________________________________

Email:____________________________________

Signature:_________________________________

Itemized Scores
Problem 1:
Problem 2:
Problem 3:
Problem 4:
Total:

(30 pts) **1.** Consider the CT signal $x(t) = \frac{sin(2\pi t)}{t}$. Let $T = 0.2$.
a) Sketch the graph of the CTFT $X(f)$ of $x(t)$. (No justification needed.)

b) Sketch the graph of the CTFT $X_s(f)$ of the signal $x_s(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$. (No justification needed)

b) Sketch the graph of the DTFT $\mathcal{X}_d(\omega)$ of the signal $x_d[n] = x(Tn)$. (No justification needed)

(15 pts) **2.** Recall the expression of the Whitaker-Kotelnikov-Shannon expansion

$$x_r(t) = \sum_{k=-\infty}^{\infty} x(kT)\ \mathrm{sinc}\ \left(\frac{t-kT}{T}\right)$$

Show (mathematically) that, for any integer n,

$$x_r(nT) = x(nT).$$

Note: You will automatically get 3 pts if you do not attempt to solve this question.

(15 pts) **3.** Give a proof for the following theorem:

Theorem: Let $x(t)$ be a signal whose CTFT $X(f)$ satisfies $X(f) = 0$ for all f such that $|f| > f_m > 0$. Let $y(t) = \text{comb}_T\{x(t)\}$ with $T < \frac{1}{2f_m}$. Then $x(t)$ can be recovered from $y(t)$.

Note: You will automatically get 3 pts if you do not attempt to solve this question.

(15 pts) **4.** Let N be the fundamental period of the discrete-time signal $x[n] = e^{j\frac{\pi}{3}n}\cos(\frac{\pi}{6}n)$. Compute the N-point DFT of x[n]. (Justify all your steps.)

Table

CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad (1)$$

$$\text{Inverse F.T.: } x(t) = \int_{-\infty}^{\infty} \mathcal{X}(f)e^{j2\pi ft}df \quad (2)$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (3)$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \quad (4)$$

Discrete Fourier Transform

Let $x[n]$ be a periodic discrete-time signal with period N

$$\text{D.F.T.: } X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn} \quad (5)$$

$$\text{Inverse D.F.T.: } x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{j\frac{2\pi}{N}kn} \quad (6)$$

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n} \quad (7)$$

-SCRATCH -
(will not be graded)