$$x[n] = \sin \omega_0 n = \frac{1}{2j} e^{j\frac{2\pi}{N}n} - \frac{1}{2j} e^{-j\frac{2\pi}{N}n}$$

Look at just one complex exponential

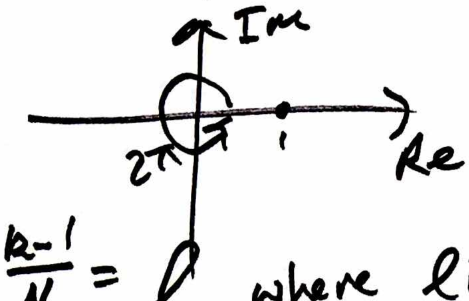
$$x_1[n] = \frac{1}{2j} e^{j\frac{2\pi}{N}n}$$

$$b_k = \frac{1}{N} \sum_{n=\langle N \rangle} x_1[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$\frac{1}{2j}\left(= \frac{1}{N} \sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}n} e^{-j\frac{2\pi}{N}kn} = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}(k-1)n} \right)$$

$$= \frac{1}{N\cdot 2j} \frac{1 - e^{-j\frac{2\pi}{N}(k-1)N}}{1 - e^{-j\frac{2\pi}{N}(k-1)}} = \frac{1}{N\cdot 2j} \frac{1 - e^{-j2\pi(k-1)}}{1 - e^{-j\frac{2\pi}{N}(k-1)}} \quad \begin{matrix} (a) \\ (b) \end{matrix}$$

(a) $1 - \underbrace{e^{-j2\pi(k-1)}}_{=1} = 0$



(b) $1 - e^{-j\frac{2\pi}{N}(k-1)} = 0$ if $\frac{k-1}{N} = l$ where l is an integer

$k = lN + 1$

IF (a) and (b) are 0, the output is $\frac{1}{2j} \cdot \frac{1}{N} \cdot N = \frac{1}{2j}$

$$b_k = \begin{cases} \frac{1}{2j} & k = lN+1 \\ 0 & \text{else} \end{cases}$$ where l is an integer ($l \in \mathbb{Z}$)

$$X \quad b_k = \begin{cases} \frac{1}{2j} & k=1 \\ 0 & \text{else} \end{cases}$$

$$b_k = \begin{cases} \frac{1}{2j} & k=1 \\ 0 & , k=0 \text{ or } k=2,3,4,\dots,N-1 \end{cases}$$

b_k is periodic with period N

Explanation

(b) $=0$ if $\frac{k-1}{N}=l$

$$\frac{1}{2j}\cdot\frac{1}{N}\sum_{k=0}^{N-1}\underbrace{e^{-j2\pi l n}}_{=1} = \frac{1}{2j}\,\frac{1}{N}\cdot N$$

Using FS to analyze LTI Systems

From chap 2:

CT: $S_c\{e^{j\omega_0 t}\} = \lambda_c e^{j\omega_0 t}$

DT: $S_D\{e^{j\omega_0 n}\} = \lambda_D e^{j\omega_0 n}$

Complex exponentials are eigenfunctions for LTI systems.

The eigenvalues are related to the impulse response:

CT: $H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t}\, dt$

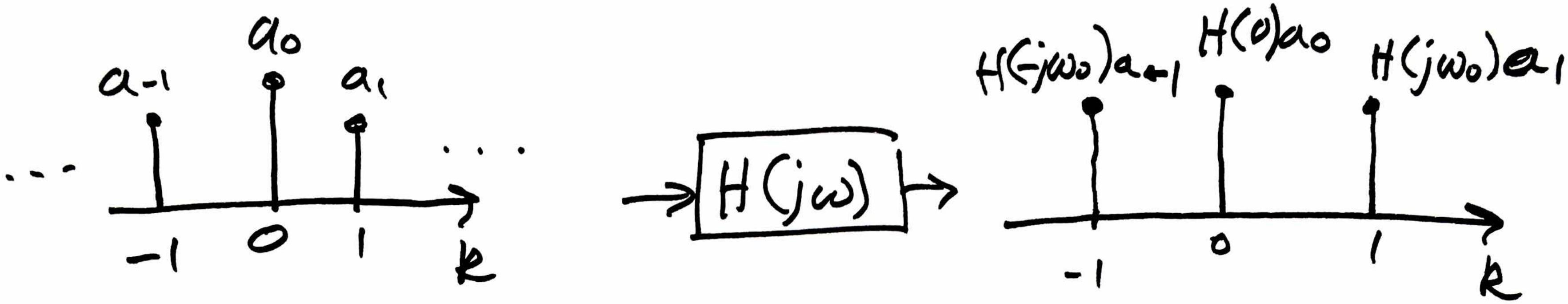
DT: $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

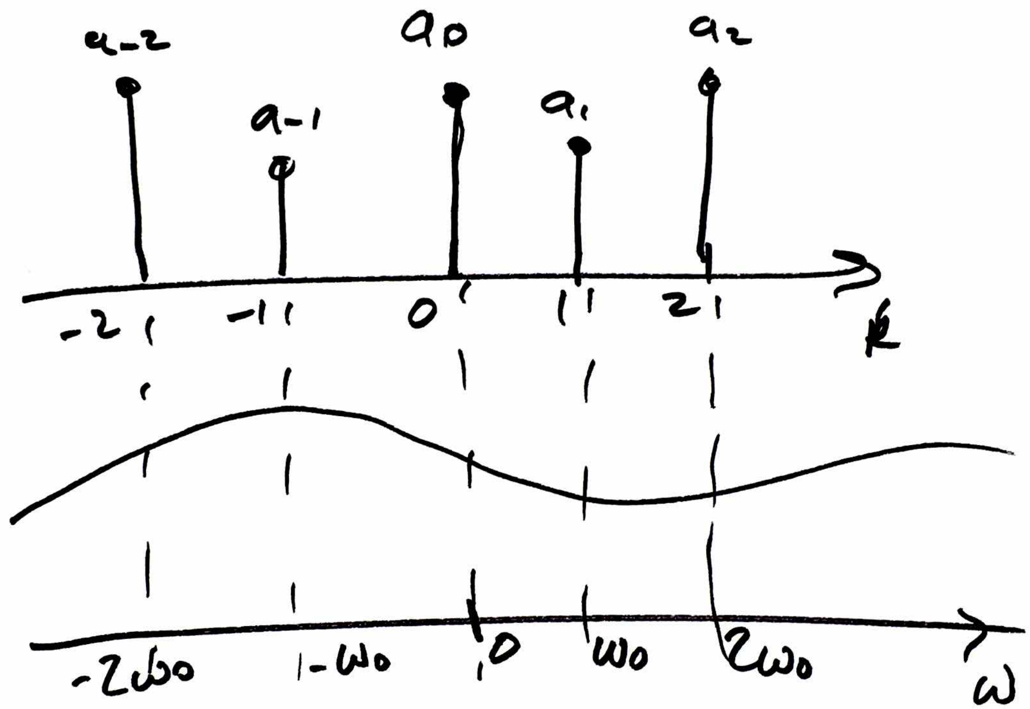
frequency response

Given the frequency response

$$S_c\{e^{j\omega_0 t}\} = H(j\omega_0)\, e^{j\omega_0 t}$$

$$S_D\{e^{j\omega_0 n}\} = H(e^{j\omega_0})\, e^{j\omega_0 n}$$





Pointwise multiplying the FS coefficients

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 t}$$

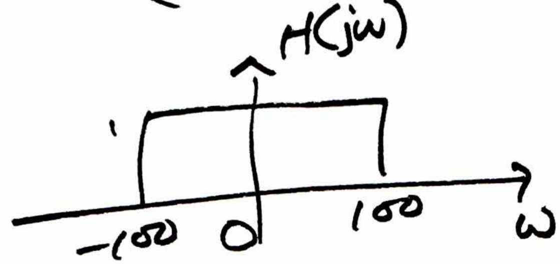
$$y(t) = S\{x(t)\} = S\left\{\sum_{k=-\infty}^{\infty} a_k e^{j\omega_0 k t}\right\}$$

$$= \sum_{k=-\infty}^{\infty} a_k S\{e^{j\omega_0 t}\} = \sum_{k=-\infty}^{\infty} a_k H(j\omega_0 k) e^{j\omega_0 k t}$$

Passing $x(t)$ into a system modifies the FS coefficients through multiplication.

OW 3.15

$$H(j\omega) = \begin{cases} 1 & |\omega| \le 100 \\ 0 & |\omega| > 100 \end{cases}$$ ideal lowpass filter



$x(t)$ with period $\frac{\pi}{6}$

$$x(t) \longrightarrow \boxed{H(j\omega)} \longrightarrow \tilde{x}(t)$$

~~a_k~~ For which k values does $a_k = 0$?

a_k weight $e^{j\omega_0 k t}$ $\quad \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{\frac{\pi}{6}} = 12$

$= e^{j12kt}$

$a_k = 0$ if $12|k| > 100$

$|k| > \frac{100}{12} = \frac{25}{3} = 8\frac{1}{3}$

$\Rightarrow |k| \ge 9, \; a_k = 0$