# Final Examination
## ECE 438
### Fall 2010
### Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.
2. You have 120 minutes to complete the 6 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.
3. This exam contains 14 pages. The last two pages can be used as scratch paper. Pages 10-12 contain a table of formulas and properties. You may tear out the scratch paper and the table **once the exam begins**. You can use any fact contained in the table without justification. If you use a non-trivial fact that is not contained in the table, you must justify it in order to get full credit.
4. This is a closed book exam. The use of any electronic device is strictly forbidden.

Name:________________________________________

Email:________________________________________

Signature:________________________________________

| **Itemized Scores** | |
|---|---|
| Problem 1: | |
| Problem 2: | |
| Problem 3: | |
| Problem 4: | |
| Problem 5: | |
| Problem 6: | |
| Total: | |

**1.** Let $x(t)$ be the continuous-time signal defined by

$$x(t) = e^{j8\pi t}\frac{\sin(8\pi t)}{\pi t}$$

(10 pts) a) Sketch the graph of the CT Fourier transform $X(f)$ of this signal.

(15 pts) b) Sketch the graph of the DT Fourier transform $X_d(\omega)$ of the sampling $x_d[n] = x(\frac{n}{10})$.

(20 pts) c) Can one construct the signal $y[n] = x(\frac{n}{20})$ directly from $x[n]$ without reconstructing $x(t)$? (yes/no) If so, explain how (no justification needed). If not, carefully explain why.

(25 pts) **2.** Write a simple mathematical expression describing the relationship between the CSFT of a continuous-space signal $f(x, y)$ and the CSFT of the signal

$$g(x, y) = \sum_{k,m=-\infty}^{\infty} f(kT_x, mT_y)\delta(x - kT_x, y - mT_y).$$

(No justification needed.)
Use your answer to sketch the CSFT of $g(x, y)$ for $f(x, y) = \frac{\sin 8\pi x}{\pi x}\frac{\sin 8\pi y}{\pi y}$, with $T_x = \frac{1}{10}$ and $T_y = \frac{1}{7}$.

(15 pts) **3.** Consider a length $N$ signal $x[n]$, with $x[n] \neq 0$ only for $n = 0, \ldots, N-1$. Let $y[n]$ be a new signal defined by padding $x[n]$ with zeros to make it length $M > N$:

$$y[n] = \begin{cases} x[n], & n = 0, \ldots, N-1 \\ 0, & n = N, \ldots, M-1. \end{cases}$$

Can one recover $x[n]$ from the M-point DFT of $y[n]$? (Yes/No) If so, write an equation describing the relationship between x[n] and the M-point DFT of $y[n]$. (No justification needed.) If not, briefly explain why.

**4.** Consider the discrete-space system defined by the equation $g[m,n] = h[m,n] ** f[m,n]$, where $h[m,n]$ is

$h[m,n] =$

| $n \backslash m$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $1$ | $\frac{-1}{4}$ | $\frac{1}{2}$ | $\frac{-1}{4}$ |
| $0$ | $\frac{-1}{4}$ | $\frac{1}{2}$ | $\frac{-1}{4}$ |
| $-1$ | $\frac{-1}{4}$ | $\frac{1}{2}$ | $\frac{-1}{4}$ |

(15 pts) a) What is the system's response to the following input (using symmetric boundary conditions)?

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

(15 pts) b) Obtain the discrete-space Fourier transform $H(\omega_1, \omega_2)$ of $h[m,n]$ and express it in terms of sines/cosines.

(20 pts) **5.** Compute the inverse z-transform of

$$X(z) = \ln(1 - z), \text{ ROC } |z| < 1.$$

Hint: Observe that $\frac{d \ln(1-z)}{dz} = \frac{-1}{1-z}$.

(30 pts) **6.** Imagine that a friend who just took EE301 this semester would like to know whether Neil Armstrong said “for a man” or “for man” when he stepped on the moon. White a short text explaining to this friend how one can use the concept of formants to answer this question. Do not assume any knowledge beyond ECE301. In other words, explain/summarize all the concepts not covered in ECE301 that you use in your explanation. You do not need to write a novel. Focus on the essential, but make sure not to forget any step. (There is extra space on the following page to continue writing your answer if you need.)

Extra space to answer Question 6.

## Table

### CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \quad (1)$$

$$\text{Inverse F.T.: } x(t) = \int_{-\infty}^{\infty} \mathcal{X}(f)e^{j2\pi ft}df \quad (2)$$

### Properties of CT Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $\mathcal{X}(f)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $\mathcal{Y}(f)$ its Fourier transform.

| | *Signal* | *FT* | |
|---|---|---|---|
| Linearity: | $ax(t) + by(t)$ | $a\mathcal{X}(f) + b\mathcal{Y}(f)$ | (3) |
| Time Shifting: | $x(t - t_0)$ | $e^{-j\omega t_0}\mathcal{X}(f)$ | (4) |
| Frequency Shifting: | $e^{j2\pi f_0 t}x(t)$ | $\mathcal{X}(f - f_0)$ | (5) |
| Duality | $\mathcal{X}(t)$ | $x(-f)$ | (6) |
| Time and Frequency Scaling: | $x(\frac{t}{a})$ | $\|a\|X\left(af\right)$ | (7) |
| Multiplication: | $x(t)y(t)$ | $\mathcal{X}(f) * \mathcal{Y}(f)$ | (8) |
| Convolution: | $x(t) * y(t)$ | $\mathcal{X}(f)\mathcal{Y}(f)$ | (9) |
| Transform of periodic signals | $\text{rep}_T[x(t)]$ | $\frac{1}{T}\text{comb}_{\frac{1}{T}}[\mathcal{X}(f)]$ | (10) |
| Transform of sampled signals | $\text{comb}_T[x(t)]$ | $\text{rep}_{\frac{1}{T}}[\mathcal{X}(f)]$ | (11) |
| | | | (12) |

### Some CT Fourier Transform Pairs

$$e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f - f_0) \quad (13)$$

$$1 \xrightarrow{\mathcal{F}} \delta(f) \quad (14)$$

$$\frac{\sin(2\pi f_0 t)}{\pi t} \xrightarrow{\mathcal{F}} u(f + f_0) - u(f - f0) = \text{rect}\left(\frac{f}{2f_0}\right) \quad (15)$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1 \quad (16)$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \xrightarrow{\mathcal{F}} \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T}) \quad (17)$$

## DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \tag{18}$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \tag{19}$$

## Properties of DT Fourier Transform

Let $x(t)$ be a signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

| | *Signal* | *F.T.* | |
|---|---|---|---|
| Linearity: | $ax[n] + by[n]$ | $a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$ | (20) |
| Time Shifting: | $x[n - n_0]$ | $e^{-j\omega n_0}\mathcal{X}(\omega)$ | (21) |
| Frequency Shifting: | $e^{j\omega_0 n}x[n]$ | $\mathcal{X}(\omega - \omega_0)$ | (22) |
| Time Reversal: | $x[-n]$ | $\mathcal{X}(-\omega)$ | (23) |
| Multiplication: | $x[n]y[n]$ | $\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$ | (24) |
| Convolution: | $x[n] * y[n]$ | $\mathcal{X}(\omega)\mathcal{Y}(\omega)$ | (25) |
| Differencing in Time: | $x[n] - x[n-1]$ | $(1 - e^{-j\omega})\mathcal{X}(\omega)$ | (26) |

## Some DT Fourier Transform Pairs

$$\sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \xrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N}) \tag{27}$$

$$e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l) \tag{28}$$

$$1 \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) \tag{29}$$

$$\frac{\sin Wn}{\pi n}, 0 < W < \pi \xrightarrow{\mathcal{F}} \mathcal{X}(\omega) = \begin{cases} 1, & 0 \le |\omega| < W \\ 0, & \pi \ge |\omega| > W \end{cases} \tag{30}$$

$\mathcal{X}(\omega)$periodic with period $2\pi$

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \tag{31}$$

$$u[n] \xrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) \tag{32}$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}} \tag{33}$$

$$(n+1)\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1 - \alpha e^{-j\omega})^2} \tag{34}$$

## Discrete Fourier Transform

Let $x[n]$ be a periodic signal with period N.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad (35)$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad (36)$$

## Properties of the discrete Fourier transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X[k]$, $X_1[k]$ and $X_2[k]$ their respective DFTs.

| | Signal | L.T. | |
|---|---|---|---|
| Linearity: | $ax_1[n] + bx_2[n]$ | $aX_1[k] + bX_2[k]$ | (37) |
| Time Shifting: | $x[n - n_0)$ | $e^{-j\frac{2\pi}{N}kn} X[k]$ | (38) |
| modulation | $e^{j\frac{2\pi}{N}kn} x[n]$ | $X[k - k_0]$ | (39) |
| Reciprocity | $X[n]$ | $Nx[-k]$ | (40) |
| | | | (41) |

## z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad (42)$$

## Properties of z-Transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X(z)$, $X_1(z)$ and $X_2(z)$ their respective z-transform. Let $R$ be the ROC of $X(z)$, let $R_1$ be the ROC of $X_1(z)$ and let $R_2$ be the ROC of $X_2(z)$.

| | Signal | z-T. | ROC | |
|---|---|---|---|---|
| Linearity: | $ax_1[n] + bx_2[n]$ | $aX_1(z) + bX_2(z)$ | At least $R_1 \cap R_2$ | (43) |
| Time Shifting: | $x[n - n_0]$ | $z^{-n_0} X(z)$ | $R$, but perhaps adding/deleting $z = 0$ | (44) |
| Time Shifting: | $x[-n]$ | $X(z^{-1})$ | $R^{-1}$ | (45) |
| Scaling in z: | $e^{j\omega_0 n} x[n]$ | $X(e^{-j\omega_0} z)$ | $R$ | (46) |
| Conjugation: | $x^*[n]$ | $X^*(z^*)$ | $R$ | (47) |
| Convolution: | $x_1[n] * x_2[n]$ | $X_1(z)X_2(z)$ | At least $R_1 \cap R_2$ | (48) |

## Continuous-space Fourier Transform

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy \quad (49)$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} dudv \quad (50)$$

-SCRATCH -
(will not be graded)

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(will not be graded)