1. Can you find a function from $[0,1] \to \mathbb{R}$ which has infinitely many discontinuities but is still Riemann integrable?

2. Suppose $f : [0,1] \to \mathbb{R}$ is a function such that for all $a, b\{x : a \leq f(x) < b\}$ is a union of disjoint intervals

   (a) Show that f is Riemann integrable.

   (b) Let $m\{x : k/N \leq f(x) < k/N\}$ be the sum of the lengths of the disjoint intervals of $\{x : k/N \leq f(x) < k/N\}$ (for example, $m([1/2, 3/4] \cup [7/8, 1)) = 1/4 + 1/8$.) Show

   $$\sum_{k=0}^{N^2} \frac{k}{N} m\{x : k/N \leq f(x) < k/N\} \to \int f$$

   as $N \to \infty$.