ECE 302 Homework 4
Due July 12, 2016

Reading assignment: Review reading assignment from Homework 4; chapter 3 section 3.4, 3.5; chapter 4 sections 4.4, 4.7.

1. Let V and I denote random variables representing the voltage and current across a (nonrandom) resitance R. Show that the average power across the resistor is equal to the product of the RMS voltage and the RMS current.

2. A binary transmission system transmits a signal X that is either -1 or 1. The received signal is $Y = X + N$ where noise N has a zero-mean Gaussian distribution with variance σ^2. Assume that $\Pr(X = -1) = 2\Pr(X = 1)$.

   (a) Find the conditional pdf of Y given the input value: $f_Y(y|X = 1)$ and $f_Y(y|X = -1)$.

   (b) Find the pdf of Y.

   (c) Show that the statement

   $$f_Y(y|X = 1)\Pr(X = 1) > f_Y(y|X = -1)\Pr(X = -1)$$

   is equivalent to $y > T$ for some suitable T. Find the value of T.

3. A coin with probability p of coming up heads is flipped continually until both heads and tails have appeared. Assume all coin flips are independent. Let the random variable X be the number of flips. Use total expectation to find the mean of X.

4. A Gaussian random voltage X volts is input to a half-wave rectifier and the output voltage is $Y = Xu(X)$ volts, where $u(x)$ s the unit step function. Assume X has mean 0 V and variance σ^2 V^2. The output voltage Y is then applied across a (nonrandom) resistance of R ohms. Your answers should be expressed in terms of the Φ or Q functions or in closed form (no integrals).

   (a) Find the probability that the current which flows through the resistor exceeds 1 Amp.

   (b) Find the probability that the power which is dissipated in the resistor exceeds 1 watt.

   (c) Find the mean and variance of the current which flows through the resistor.

   (d) Find the mean and variance of the power which is dissipated in the resistor.

5. The random variable X is uniformly distributed in the interval $[0, a]$. Suppose a is unknown, so we estimte a by the maximum value observed in n independent repetitions of the experiment; that is, we estimate a by $Y = \max\{X_1, X_2, ...X_n\}$, where the X_i are distributed as X.

   (a) Find $\Pr(Y \leq y)$.

   (b) Find the pdf of Y.

   (c) Find the mean and variance of Y, and explain why Y is a good estimate for a when n is large.

6. (a) Find the characteristic function $\varphi_X(\omega)$ of a uniform random variable in $[-a, a]$, where $a > 0$.

   (b) Find the characteristic function $\varphi_X(\omega)$ of an exponential random variable with mean $1/\lambda$, where $\lambda > 0$. Use table 4.1 in the text to find the distribution of a random variable with characteristic function $\varphi_X^n(\omega)$, where $n > 0$ is an integer.

   (c) Use the moment theorem to find the n^{th} moment of the random variable from part (b) for $n \geq 1$.