Final Examination
ECE 438
Fall 2011
Instructor: Prof. Mimi Boutin

Instructions:

1. Wait for the "BEGIN" signal before opening this booklet. In the meantime, read the instructions below and fill out the requested info.

2. You have 120 minutes to complete the 6 questions contained in this exam. **When the end of the exam is announced, you must stop writing immediately**. Anyone caught writing after the exam is over will get a grade of zero.

3. This exam contains 12 pages. The last two pages can be used as scratch paper. Pages 8-10 contain a table of formulas and properties. You may tear out the scratch paper and the table **once the exam begins**. You can use any fact contained in the table without justification. If you use a non-trivial fact that is not contained in the table, you must justify it in order to get full credit.

4. This is a closed book exam. The use of any electronic device is strictly forbidden. All electronic devices must be turned off and stored away, out of reach and out of sight (e.g., not in your pocket).

Name:______________________________

Email:______________________________

Signature:____________________________

Itemized Scores
Problem 1:
Problem 2:
Problem 3:
Problem 4:
Problem 5:
Problem 6:
Total:

1. (40 pts) Let $x(t)$ be the continuous-time signal defined by

$$x(t) = \frac{\cos(10\pi t)\sin(4\pi t)}{\pi t}$$

Let $x_1[n] = x(\frac{n}{50})$ and $x_2[n] = x(\frac{n}{23})$

a) Sketch the graph of the DTFT of $x_1[n]$. (No justification needed)

b) Sketcn the graph of the DTFT of $x_2[n]$. (No justification needed)

c) Sketch the graph of the DTFT of $x_3[n] = x_1[2n]$. (No justification needed)

2. (20 pts) Use the definition of the z-transform (i.e. the summation formula) to obtain the inverse z-transform of

$$X(z) = \frac{1}{z^2 - 5z + 6}, \text{ ROC: } 2 < |z| < 3.$$

(Justify *all* steps of your computation.)

3. (15 pts) Joe is a student in ECE301 who is learning the DTFT. He decides to experiment a little bit with this concept using MATLAB. He begins by building a discrete-time signal as follows:

```
delta=0.00001
for n=0:1:1000000
y[n]=sin( 2*pi*440*n*delta)
end
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Joe thinks that, since y[n] represents a pure frequency, then its DTFT $\mathcal{Y}(\omega)$ should consist of two impulses, repeated periodically every 2π. To confirm this, he computes the value of $\mathcal{Y}(\omega)$ at 10,000 sample points (using the definition of the DTFT, i.e., the summation formula) and plots the results on a graph. To his surprise, there are ripples around the location of the impulses. Explain (briefly) why there are ripples around the impulses (i.e., what do the ripples correspond to in the time domain?) .

4. (15 points) A phoneme is pronounced with a pitch frequency of 125Hz. The phoneme has two formants: one at 500 Hz, and one at 1 kHz. This signal is sampled at 3kHz. Sketch the approximate location of the poles of the transfer function $H(z)$ corresponding to the vocal tract position for this phoneme. (No justification needed.)

5. (15 pts) When one zero pads a signal to increase its duration (i.e., adding zero signal values at the end of the signal), what effect does this have on its DFT? (Give a short and precise answer and then prove it)

6. (20 its) A discrete-space system is described by the equation $g[m,n] = f[m,n] * *h[m,n]$ where

$$h[m,n] = \begin{array}{|c|c|c|} \hline -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \\ \hline -\frac{1}{9} & \frac{8}{9} & -\frac{1}{9} \\ \hline -\frac{1}{9} & -\frac{1}{9} & -\frac{1}{9} \\ \hline \end{array} \begin{array}{c} 1 \\ 0 \; n \\ -1 \end{array}$$

$$-1 \quad 0 \quad 1$$

$$m$$

a) Compute the frequency response of this system. (Write the intermediate steps of your computation and simplify your answer as much as possible. It should be a simple function of sine/cosine at the end).

b) What are the characteristics of this filter?

Table

CT Fourier Transform

$$\text{F.T. : } \mathcal{X}(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \tag{1}$$

$$\text{Inverse F.T.: } x(t) = \int_{-\infty}^{\infty} \mathcal{X}(f)e^{j2\pi ft}df \tag{2}$$

Properties of CT Fourier Transform

Let $x(t)$ be a continuous-time signal and denote by $\mathcal{X}(f)$ its Fourier transform. Let $y(t)$ be another continuous-time signal and denote by $\mathcal{Y}(f)$ its Fourier transform.

	Signal	*FT*	
Linearity:	$ax(t) + by(t)$	$a\mathcal{X}(f) + b\mathcal{Y}(f)$	(3)
Time Shifting:	$x(t - t_0)$	$e^{-j\omega t_0}\mathcal{X}(f)$	(4)
Frequency Shifting:	$e^{j2\pi f_0 t}x(t)$	$\mathcal{X}(f - f_0)$	(5)
Duality	$\mathcal{X}(t)$	$x(-f)$	(6)
Time and Frequency Scaling:	$x(\frac{t}{a})$	$\|a\|X\,(af)$	(7)
Multiplication:	$x(t)y(t)$	$\mathcal{X}(f) * \mathcal{Y}(f)$	(8)
Convolution:	$x(t) * y(t)$	$\mathcal{X}(f)\mathcal{Y}(f)$	(9)
Transform of periodic signals	$\text{rep}_T[x(t)]$	$\frac{1}{T}\text{comb}_{\frac{1}{T}}[\mathcal{X}(f)]$	(10)
Transform of sampled signals	$\text{comb}_T[x(t)]$	$\text{rep}_{\frac{1}{T}}[\mathcal{X}(f)]$	(11)
			(12)

Some CT Fourier Transform Pairs

$$e^{j2\pi f_0 t} \xrightarrow{\mathcal{F}} \delta(f - f_0) \tag{13}$$

$$1 \xrightarrow{\mathcal{F}} \delta(f) \tag{14}$$

$$\frac{\sin(2\pi f_0 t)}{\pi t} \xrightarrow{\mathcal{F}} u(f + f_0) - u(f - f0) = \text{rect}\left(\frac{f}{2f_0}\right) \tag{15}$$

$$\delta(t) \xrightarrow{\mathcal{F}} 1 \tag{16}$$

$$\sum_{n=-\infty}^{\infty} \delta(t - nT) \xrightarrow{\mathcal{F}} \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta(f - \frac{k}{T}) \tag{17}$$

DT Fourier Transform

Let $x[n]$ be a discrete-time signal and denote by $X(\omega)$ its Fourier transform.

$$\text{F.T.:}\mathcal{X}(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \quad (18)$$

$$\text{Inverse F.T.: } x[n] = \frac{1}{2\pi}\int_{2\pi} \mathcal{X}(\omega)e^{j\omega n}d\omega \quad (19)$$

Properties of DT Fourier Transform

Let $x(t)$ be a signal and denote by $\mathcal{X}(\omega)$ its Fourier transform. Let $y(t)$ be another signal and denote by $\mathcal{Y}(\omega)$ its Fourier transform.

	Signal	*F.T.*	
Linearity:	$ax[n] + by[n]$	$a\mathcal{X}(\omega) + b\mathcal{Y}(\omega)$	(20)
Time Shifting:	$x[n - n_0]$	$e^{-j\omega n_0}\mathcal{X}(\omega)$	(21)
Frequency Shifting:	$e^{j\omega_0 n}x[n]$	$\mathcal{X}(\omega - \omega_0)$	(22)
Time Reversal:	$x[-n]$	$\mathcal{X}(-\omega)$	(23)
Multiplication:	$x[n]y[n]$	$\frac{1}{2\pi}\mathcal{X}(\omega) * \mathcal{Y}(\omega)$	(24)
Convolution:	$x[n] * y[n]$	$\mathcal{X}(\omega)\mathcal{Y}(\omega)$	(25)
Differencing in Time:	$x[n] - x[n-1]$	$(1 - e^{-j\omega})\mathcal{X}(\omega)$	(26)

Some DT Fourier Transform Pairs

$$\sum_{k=0}^{N-1} a_k e^{jk(\frac{2\pi}{N})n} \xrightarrow{\mathcal{F}} 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - \frac{2\pi k}{N}) \quad (27)$$

$$e^{j\omega_0 n} \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi l) \quad (28)$$

$$1 \xrightarrow{\mathcal{F}} 2\pi \sum_{l=-\infty}^{\infty} \delta(\omega - 2\pi l) \quad (29)$$

$$\frac{\sin Wn}{\pi n}, 0 < W < \pi \xrightarrow{\mathcal{F}} \mathcal{X}(\omega) = \begin{cases} 1, & 0 \le |\omega| < W \\ 0, & \pi \ge |\omega| > W \end{cases} \quad (30)$$

$\mathcal{X}(\omega)$periodic with period 2π

$$\delta[n] \xrightarrow{\mathcal{F}} 1 \quad (31)$$

$$u[n] \xrightarrow{\mathcal{F}} \frac{1}{1 - e^{-j\omega}} + \pi \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k) \quad (32)$$

$$\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{1 - \alpha e^{-j\omega}} \quad (33)$$

$$(n+1)\alpha^n u[n], |\alpha| < 1 \xrightarrow{\mathcal{F}} \frac{1}{(1 - \alpha e^{-j\omega})^2} \quad (34)$$

Discrete Fourier Transform

Let $x[n]$ be a periodic signal with period N.

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi}{N}kn} \quad (35)$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi}{N}kn} \quad (36)$$

Properties of the discrete Fourier transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X[k]$, $X_1[k]$ and $X_2[k]$ their respective DFTs.

	Signal	L.T.	
Linearity:	$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$	(37)
Time Shifting:	$x[n-n_0)$	$e^{-j\frac{2\pi}{N}kn}X[k]$	(38)
modulation	$e^{j\frac{2\pi}{N}kn}x[n]$	$X[k-k_0]$	(39)
Reciprocity	$X[n]$	$Nx[-k]$	(40)
			(41)

z-Transform

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad (42)$$

Properties of z-Transform

Let $x[n]$, $x_1[n]$ and $x_2[n]$ be three DT signals and denote by $X(z)$, $X_1(z)$ and $X_2(z)$ their respective z-transform. Let R be the ROC of $X(z)$, let R_1 be the ROC of $X_1(z)$ and let R_2 be the ROC of $X_2(z)$.

	Signal	z-T.	ROC	
Linearity:	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least $R_1 \cap R_2$	(43)
Time Shifting:	$x[n-n_0]$	$z^{-n_0}X(z)$	R, but perhaps adding/deleting $z=0$	(44)
Time Shifting:	$x[-n]$	$X(z^{-1})$	R^{-1}	(45)
Scaling in z:	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R	(46)
Conjugation:	$x^*[n]$	$X^*(z^*)$	R	(47)
Convolution:	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least $R_1 \cap R_2$	(48)

Continuous-space Fourier Transform

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dxdy \quad (49)$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} dudv \quad (50)$$

-SCRATCH -
(will not be graded)

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