FS representation

$$x(t) \xleftrightarrow{FS} a_k$$

fourier series

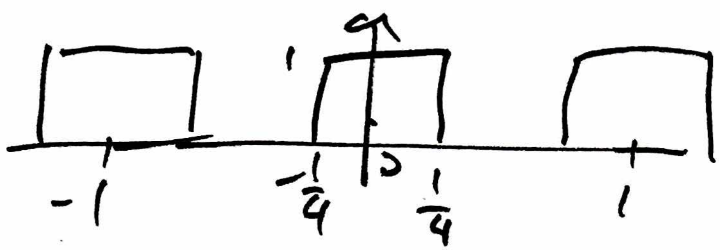
a_k: FS coefficients, spectral components

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{\infty} a_k e^{jk\frac{2\pi}{T_0} t}$$

Review

<u>Ex</u>

$$x(t) = \underbrace{\text{rep}_1}_{\text{replicates with period } 1}\Big(\underbrace{\text{rect}}_{\text{rectangle with width } \frac{1}{2}}\Big(\frac{t}{\frac{1}{2}}\Big)\Big)$$



$a_k = \frac{1}{\pi k}\sin\left(\frac{\pi}{2}k\right) \quad k \neq 0$

$a_0 = \frac{1}{2}$

$$x(t) = \frac{1}{2} + \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty} \left(\frac{1}{\pi k}\sin\left(\frac{\pi}{2}k\right)\right) e^{jk2\pi t}$$

Convergence of FS

Define $x_N(t) = \sum_{k=-N}^{N} a_k e^{jk\omega_0 t}$

$$x(t) \xleftrightarrow{FS} a_k$$

$e_N(t) = x(t) - x_N(t)$ ~ error of the truncated FS

For any continuous function

$$\int_T |e_N(t)|^2 dt = 0$$

What about $u(t)$?

- Dirichlet Conditions

1) $\int_T |x(t)|\, dt < \infty$ absolutely integrable

2) Bound variation over a period

→ finite number of maxima and minima in a period

3) Finite number of discontinuities within a period.

If met, $x_\infty(t) = x(t)$ everywhere except at discontinuities

FS Properties

$x(t) \xleftrightarrow{FS} a_k$ true for a particular T_0

- Linear

$$x_1(t) \xleftrightarrow{FS} a_k$$
$$x_2(t) \xleftrightarrow{FS} b_k$$
$$c_1 x_1(t) + c_2 x_2(t) \xleftrightarrow{FS} c_1 a_k + c_2 b_k$$

$x_1(t)$ & $x_2(t)$ have the same period

- Time shift

$$x(t) \xleftrightarrow{FS} a_k$$
$$x(t-t_0) \xleftrightarrow{FS} e^{-k\left(\frac{2\pi}{T_0}\right)t_0} a_k$$

- Time reversal

$$x(t) \xleftrightarrow{FS} a_k$$
$$x(-t) \longleftrightarrow a_{-k}$$

- Time scaling

$$x(\alpha t) \xleftrightarrow{FS} a_k$$

$$x(\alpha t) = \sum_{k=-\infty}^{\infty} a_k e^{jk(\alpha\omega_0)t}$$

- Multiplication

$$x_1(t) \xleftrightarrow{FS} a_k$$

$$x_2(t) \xleftrightarrow{FS} b_k$$

$$x_1(t)\cdot x_2(t) \xleftrightarrow{FS} \sum_{l=-\infty}^{\infty} a_l b_{k-l}$$

$x(t)$ is real

$a_2 = a + jb$

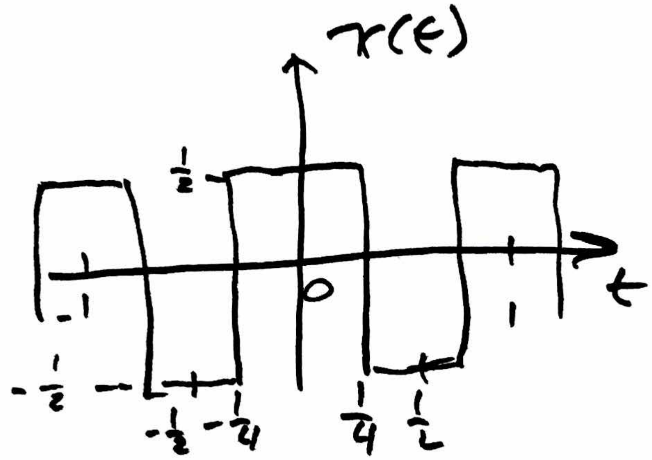
$a_{-2} = a - jb$

- Parseval's Relation

$$\frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |a_k|^2$$

* Look at page 206 when doing the homework.

Ex



$T_0 = 1$

$$a_0 = 0$$

$$a_k = \frac{1}{T_0}\int_{T_0} x(t)\, e^{-jk\frac{2\pi}{T_0}t}\, dt$$

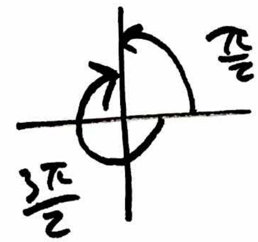
$$= 1\int_{-\frac14}^{\frac34} x(t)\, e^{-jk2\pi t}\, dt = \frac12\int_{-\frac14}^{\frac14} e^{-jk2\pi t}\, dt - \frac12\int_{\frac14}^{\frac34} e^{-jk2\pi t}\, dt$$

$$= \frac12\left(\frac{-1}{jk2\pi}\left[e^{-jk2\pi t}\right]_{-\frac14}^{\frac14} + \frac{1}{jk2\pi}\left[e^{-jk2\pi t}\right]_{\frac14}^{\frac34}\right)$$

$$= \frac12 \cdot \frac{1}{jk2\pi}\left(-e^{-jk2\pi\frac14} + e^{jk2\pi\frac14} + e^{-jk2\pi\frac34} - e^{-jk2\pi\frac14}\right)$$

$$e^{-jk\frac{3\pi}{2}}$$

$$= e^{jk\frac{\pi}{2}}$$



$$e^{-jk\frac{3\pi}{2}} \cdot \underbrace{e^{j2\pi k}}_{1} = e^{jk\frac{\pi}{2}}$$

$= 1 \ \forall k$
(for all)

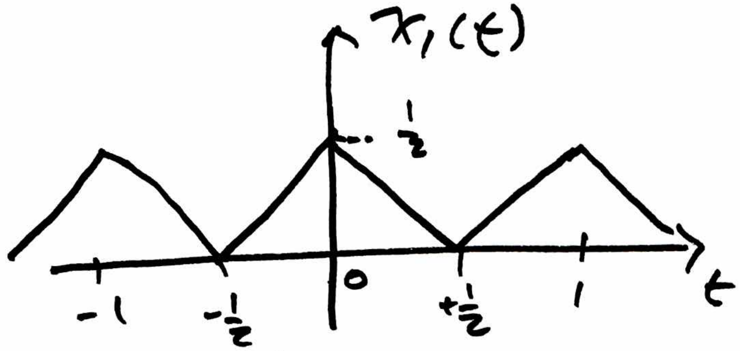
$$= \frac{1}{jk2\pi}\left(e^{jk\frac{\pi}{2}} - e^{-jk\frac{\pi}{2}}\right) = \frac{1}{k\pi}\sin\left(\frac{\pi}{2}k\right)$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Vertical shift only changes the DC term, a_0.

Sometimes easier to shift, find a_k, adjust a_0.

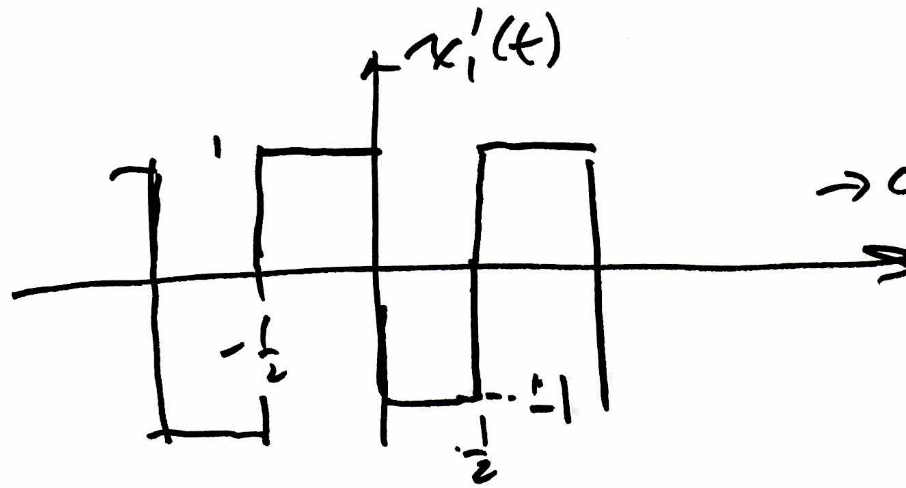
Ex



$T_0 = 1$

$$a_k = \frac{1}{1}\int_{-\frac{1}{2}}^{0}\left(t+\frac{1}{2}\right)e^{-j2\pi t\cdot k}\,dt + \frac{1}{1}\int_{0}^{\frac{1}{2}}\left(-t+\frac{1}{2}\right)e^{-j2\pi k t}\,dt$$

need integration by parts



→ can easily find the FS coefficients