Assignment 5 - Conformal Mapping & Harmonic Functions

1. Find a 1-1 conformal mapping from

$$\{\operatorname{Re} z > 0\} - [0, 1]$$

onto the unit disc (disk).

2. Suppose u and v are real-valued harmonic functions on a domain Ω.

   (a) If $u = v$ on a set with a limit point in Ω, does it follow that $u = v$ on Ω? Explain.

   (b) If u and v satisfy the Cauchy-Riemann equations on a set with a limit point in Ω, does it follow that $u + iv$ is analytic on Ω? Explain.

3. Find a 1-1 conformal mapping from the strip

$$\{z \mid 0 < \operatorname{Im} z < 1\}$$

onto the half-strip

$$\{z \mid 0 < \operatorname{Re} z,\ 0 < \operatorname{Im} z < 1\}.$$

4. Find all real-valued harmonic functions that are constant on all vertical lines.

5. Find a 1-1 conformal map from the eighth disque

$$\left\{z = re^{i\theta} \mid 0 < r < 1,\ 0 < \theta < \frac{\pi}{4}\right\}$$

onto $\{z \mid 0 < \operatorname{Im} z < 1\}$.

6. Construct a 1-1 holomorphic map from the half-disq̊

$$D = \{z : |z| < 1,\ \operatorname{Im} z > 0\}$$

onto the unit disQ̂.

7. Find all fractional linear transformations $T(z)$ which map

$$H^+ = \{z \mid \operatorname{Im} z > 0\}$$

onto the unit disk̀ue such that $T(i) = 0$.

8. Suppose that a function u, harmonic in a neighborhood of the origin, equals 0 on the real and imaginary axes. Prove that u is even.