EE 301 **Exam 3** **4/12/05**

Name: ____________________

General Instructions:

- Write your name on every page of the exam.
- Do not write on the backs of the pages. If you need more paper, it will be provided to you upon request.
- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet will be handed out.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 100 points.
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 50 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams from everyone.

This exam is for Krogmeier's section of 301.

Do not open the exam until you are told to begin.

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Problem 1. [30 pts. total] Let $x[n]$ be the length 4 discrete-time pulse defined by

$$x[n] = \begin{cases} 1 & n = 0,\ 1,\ 2,\ 3 \\ 0 & \text{otherwise} \end{cases} .$$

(a) [5 pts.] Find an expression for the discrete-time Fourier transform $X(e^{j\omega})$ of $x[n]$. Simplify as much as possible.

(b) [10 pts.] Carefully plot the magnitude and phase of $X(e^{j\omega})$ over the fundamental period $-\pi < \omega \leq \pi$. Reduce the phase so that it always lies between $-\pi$ and π.

(c) With the $X(e^{j\omega})$ from part (a), define a periodic sequence of discrete Fourier series coefficients by

$$\tilde{X}_k = \frac{1}{N} X(e^{j\omega})\bigg|_{\omega = 2\pi k/N} .$$

Let $\tilde{x}[n]$ be the periodic discrete-time sequence whose discrete-time Fourier series is $\tilde{X}_k$. Sketch a couple of periods of $\tilde{x}[n]$ for the following values of N (explain what you are doing):

(c-i) [5 pts.] $N = 7$.

(c-ii) [5 pts.] $N = 10$.

(c-iii) [5 pts.] $N = 3$.

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Problem 2. [25 pts. total] Let $x[n]$ be periodic with period N and let X_k denote its discrete Fourier series, i.e., $x[n] = x[n+N]$ for all integers n, $X_k = X_{k+N}$ for all integers k and

$$\begin{aligned} x[n] &= \sum_{k=0}^{N-1} X_k e^{j2\pi kn/N} \\ X_k &= \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N}. \end{aligned}$$

(a) [15 pts.] Note that $x[n]$ is also periodic with period $2N$. Let $\bar{X}_k$ denote its discrete Fourier series corresponding to this larger period, i.e., $\bar{X}_k = \bar{X}_{k+2N}$ for all integers k and

$$\bar{X}_k = \frac{1}{2N} \sum_{n=0}^{2N-1} x[n] e^{-j2\pi kn/(2N)}.$$

Derive a simple relationship between X_k and $\bar{X}_k$ (you must show your work).

(b) [10 pts.] Generalize the above to find the relationship between X_k and the discrete Fourier series $\tilde{X}_k$, which we get when we consider $x[n]$ to be periodic with period KN (here you can make an educated guess based upon the derivation in (a)).

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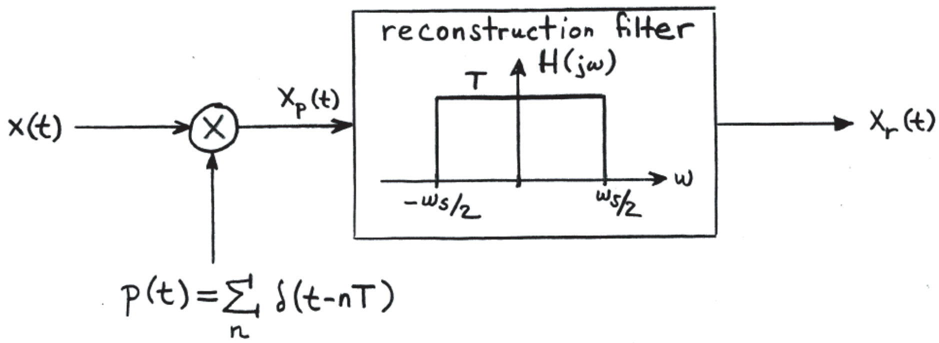
Problem 3. [45 pts. total] In this problem we will study impulse sampling and ideal low-pass filter reconstruction in the case where the signal is *not* bandlimited, resulting in some residual aliasing distortion. We will attempt to quantify the energy in the aliased terms and compare it to the energy in the desired signal at the output of the reconstruction filter. For simplicity we will use the following signal whose Fourier transform is given:

$$x(t) = \frac{2}{1+t^2} \leftrightarrow 2\pi e^{-|\omega|} = X(j\omega).$$

The sampler-reconstruction block diagram is shown below; T is the sample spacing and $\omega_s = 2\pi/T$ is the radian sampling frequency.



(a) [10 pts.] In class we derived the following relationship between the Fourier transforms of $x(t)$ and $x_p(t)$:

$$X_p(j\omega) = \frac{1}{T}\sum_k X(j(\omega - k\omega_s)).$$

Rederive this general result using basic Fourier transform pairs and properties found in the formula sheet.

(b) Make careful plots of $X_p(j\omega)$ for the signal $x(t) = 2/(1+t^2)$ above for the two sampling frequencies:

(b-i) [3 pts.] ω_s such that $e^{-\omega_s} = 0.01$.

(b-ii) [3 pts.] ω_s such that $e^{-\omega_s} = 0.25$

Which sampling frequency results in smaller aliasing distortion?

(c) [7 pts.] The output of the reconstruction filter $x_r(t)$ has a Fourier transform, which can be written

$$\begin{aligned} X_r(j\omega) &= X_p(j\omega)H(j\omega) \\ &= \begin{cases} \sum_k X(j(\omega - k\omega_s)) & -\omega_s/2 \le \omega \le \omega_s/2 \\ 0 & \text{otherwise} \end{cases}. \end{aligned}$$

Thus, for $-\omega_s/2 \le \omega \le \omega_s/2$, the spectrum $X_r(j\omega)$ can be written as the sum of two parts:

$$X_r(j\omega) = X_{desired}(j\omega) + X_{aliased}(j\omega).$$

In terms of $X(j\omega) = 2\pi e^{-|\omega|}$, give expressions for $X_{desired}(j\omega)$ and $X_{aliased}(j\omega)$.

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(d) [10 pts.] For values of ω such that $-\omega_s/2 \leq \omega \leq \omega_s/2$, sum the series to give a closed form expression for $X_{aliased}(j\omega)$.

(e) [6 pts.] For ω_s such that $e^{-\omega_s} = 0.01$, plot $X_{desired}(j\omega)$ and $X_{aliased}(j\omega)$ on the same axes.

(f) [6 pts.] Find expressions for the energies in the desired and aliased terms and compare.

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