EE 301 **Exam 2** **10/14/04**

Name: ______________________

General Instructions:

- The exam is closed book and closed notes. Calculators are **not** allowed or needed.
- A formula sheet is attached at the end of the exam.
- Your work must be explained to receive full credit.
- Point values for each problem are as indicated. The exam totals 100 points.
- Do all work in the blue books provided. Put your name and student identification number on the blue book. This exam problem sheet **must be handed in** with your blue book.
- All plots must be carefully drawn with axes labeled.
- If you finish the exam during the first 50 minutes, you may turn it in and leave. During the last 10 minutes you must remain seated until we pick up exams and blue books from everyone.
- This exam is for Krogmeier's section only.

Do not open the exam until you are told to begin.

1. (35 pts.) Let $x(t)$ be periodic with fundamental period T and fundamental frequency $\omega_0 = 2\pi/T$. Suppose the Fourier series coefficients of $x(t)$ are denoted by x_n.

   (a) Define a new periodic function by $\bar{x}(t) = x(t - T/2)$. Let $\bar{x}_n$ denote the Fourier series coefficients of $\bar{x}(t)$. Express $\bar{x}_n$ in terms of x_n.

   (b) Define some new Fourier series coefficients by

$$f_n = \begin{cases} x_n & \text{if } n \text{ is even.} \\ 0 & \text{if } n \text{ is odd.} \end{cases}$$

   Express the following functions in terms of $x(t)$:

   i. $f(t) = \sum_{n=-\infty}^{\infty} f_n e^{jn\omega_0 t}$.

   ii. $\tilde{f}(t) = \sum_{n=-\infty}^{\infty} f_{2n} e^{jn\omega_0 t}$.

   (c) Let

$$g_n = \begin{cases} 0 & \text{if } n \text{ is even.} \\ x_n & \text{if } n \text{ is odd.} \end{cases}$$

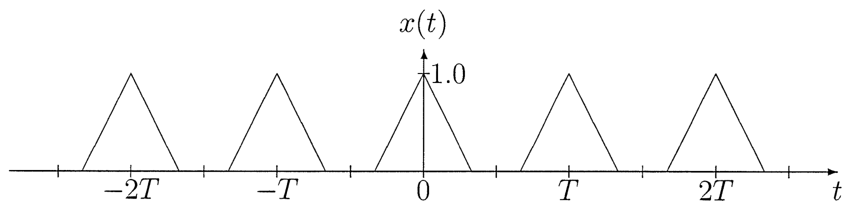
   Was a typo. Should be $\tilde{g}(t) = \sum_{n=-\infty}^{\infty} g_{2n+1} e^{jn\omega_0 t}$

   Express the following functions in terms of $x(t)$:

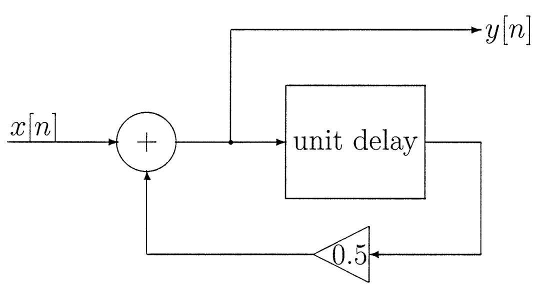
   i. $g(t) = \sum_{n=-\infty}^{\infty} g_n e^{jn\omega_0 t}$.

   ii. $\tilde{g}(t) = \sum_{n=-\infty}^{\infty} g_{2n} e^{jn\omega_0 t}$.

   (d) For the periodic triangular wave $x(t)$ shown below, plot $f(t)$, $g(t)$, $\tilde{f}(t)$, and $\tilde{g}(t)$.



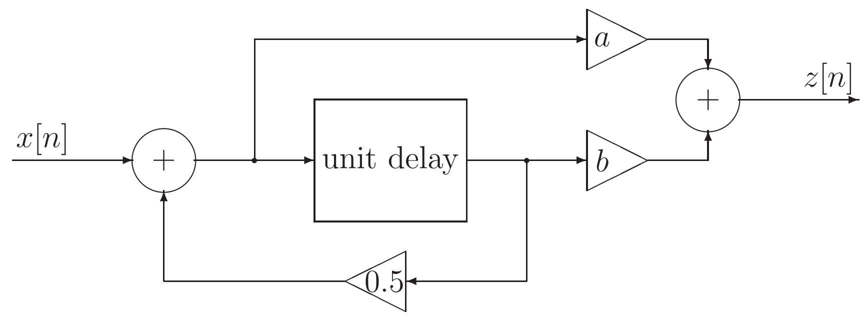
2. (30 pts.) Consider the discrete-time system below with input $x[n]$ and output $y[n]$.



   (a) From the block diagram find the difference equation relating input $x[n]$ and output $y[n]$.

   (b) Find the causal impulse response of the above system.

(c) Suppose the system above is modified as shown below. Find the new impulse response considering $z[n]$ as output and $x[n]$ as input. Your answer will depend upon the unknown multiplier gains a and b.



(d) Solve for the values of the multiplier gains a and b needed in order to make the overall impulse response equal to $\delta[n]$.

(e) Using the results above what is an inverse system for the original discrete-time system of part (a), i.e., give the impulse response of an inverse system.

3. (35 pts.) The output $y(t)$ of a causal LTI system is related to the input $x(t)$ by an integro-differential equation

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} + 5y(t) = \int_{-\infty}^{\infty} x(\tau)z(t-\tau)\mathrm{d}\tau - 2x(t)$$

where $z(t)$ is the impulse response of a sub-system.

(a) Draw a block diagram implementation of the system using a single integrator and a block for the sub-system with impulse response $z(t)$.

(b) Find the frequency response of the overall system $H(j\omega) = Y(j\omega)/X(j\omega)$ in terms of $Z(jw)$. *Hint:* Take the Fourier Transform of the overall equation.

(c) Answer the following for the case where $z(t) = e^{-2t}u(t) + \delta(t)$.

i. Find $Z(j\omega)$.

ii. Use your answer above to simplify the expression for the frequency response of the overall system $H(j\omega)$.

iii. Can the defining equation of the overall system be re-written as an ordinary differential equation? If so, give the equation. Explain your answer.

iv. Find the impulse response of the overall system.

(d) Repeat part (c) for the case where