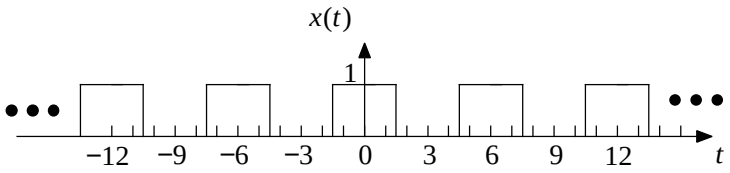
30 April 2002 **Name:**_____

EE 438 **Final Exam** **Spring 2002**

- You have 120 minutes to work the following five problems.
- Be sure to show all your work to obtain full credit.
- The exam is closed book and closed notes.
- Calculators are permitted.

1. (30 pts.) Consider the waveform $x(t)$ shown below



This signal is sampled at interval $T = 1$ to generate the signal $x[n]$.

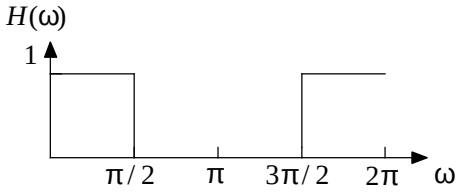
a. Suppose $x[n]$ is input to the system described by $y[n] = \frac{1}{2}(x[n] + x[n-1])$.

Plot the output signal $y[n]$.

b. Find the CTFT $X(f)$ of $x(t)$.

b. Using your answer to part b. and the relation between CTFT and DTFT, find the DTFT $X(\omega)$ of $x[n]$.

b. The signal $x[n]$ is input to an LTI system with frequency response:



Find the output signal $y[n]$.

1. (continued)

2. (30 pts) A causal LTI system has transfer function

$$H(z) = \frac{1 - z^{-1}}{1 + \frac{1}{2} z^{-2}}$$

a. Find a difference equation that can be used to implement this system.

b. Plot the poles and zeros for this system in the Z plane. Is the system BIBO stable?

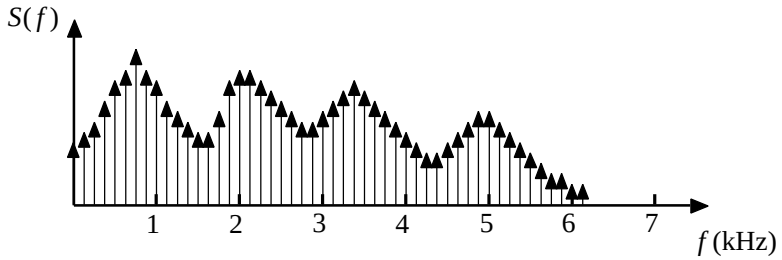
c. Use the graphical approach to find the magnitude $|H(\omega)|$ of the frequency response at the frequencies $\omega = 0, \pi/2,$ and π.

d. Use the graphical approach to find the phase $H(\omega)$ of the frequency response at the frequencies $\omega = 0$ and $\pi/2$.

e. Use ZT methods to find the response of this system to a unit step input $u[n]$.

2. (continued)

3 (30 pts.) The figure below shows the CTFT of the speech waveform $s(t)$ for a single phoneme.



a. Is this phoneme voiced or unvoiced? What is the pitch period, and what are the first three formant frequencies?
b. Sketch what a wideband spectrogram of this waveform would look like. Be sure to label all important quantities.
c. Sketch what a narrowband spectrogram of this waveform would look like. Be sure to label all important quantities.

The waveform $s(t)$ with CTFT $S(f)$ shown above is bandlimited to 4 kHz, and sampled at an 8 kHz rate. A 4096 point FFT $X[k]$ is computed of a segment of this data.

d. Sketch what this FFT would look like for $k = 0,\ldots,4095$. Be sure to indicate the interval in k corresponding to the pitch period, and the values of k corresponding to the first three formants.

3. (continued)

4. (30 pts.)

   a. Sketch the signal $f(x,y) = [1 + \cos(2\pi x)]\,\mathrm{rect}(x)$.

   b. Find and sketch the CSFT $F(u,v)$ of the signal $f(x,y)$ in part a. above.

   c. Consider a spatial filter with point spread function $h[m,n]$ given below

$h[m,n]$		n = 0	n = 1
m	0	2	−1
	1	−1	1

   d. Find a difference equation that can be used to implement this filter.

   e. Find the output $g[m,n]$ when the filter in part c. is applied to the following input image

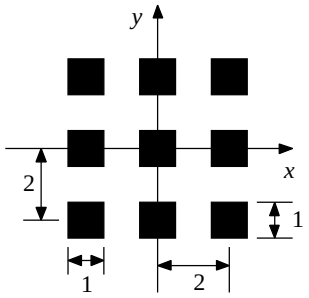
0	0	0	0	0	0	0
0	0	0	0	1	0	0
0	0	0	1	1	0	0
0	0	1	1	1	0	0
0	1	1	1	1	0	0
0	0	0	0	0	0	0
0	0	0	0	0	0	0

   f. Find a simple expression for the magnitude $|H(\mu,\nu)|$ of the frequency response of this filter, and sketch it along the μ and ν axes, and the $\mu = \nu$ axis. (*Hint:* Write $h[m,n]$ as the sum of an impulse $\delta[m,n]$ and another signal.)

4. (continued)

5. (30 pts.)

Consider the signal $f(x,y)$ shown below



which has value 1 in the shaded areas, and value 0, elsewhere.

a. Find a simple expression for the CSFT $F(u,v)$ of this signal, and sketch it.

b. Find the convolution of $f(x,y)$ with itself, and sketch it. *Note:* You can use a graphical approach to solve this problem. It is not necessary to write down a detailed expression for the answer. An accurate and complete picture will get you full credit, assuming you also indicate how you derived that picture.

c. Sketch the Radon transform $p_\theta(t)$ of $f(x,y)$ for the angles $\theta = 0, \pi/4,$ and $\pi/2$. *Note:* You can use a graphical approach to solve this problem. It is not necessary to write down a detailed expression for the answer. Accurate and complete plots will get you full credit, assuming you also indicate how you derived those plots.

5. (continued)

1. _____

2. _____

3. _____

4. _____

5. _____

Total _____