1. Let $f \in C[0,1]$ and suppose $\int_0^1 x^n f = 0$ for every $n = 0, 1, 2, \ldots$. Show $f = 0$.
2. Recall that the polynomials from $\mathbb{R}$ to $\mathbb{R}$ separate points and are nowhere vanishing; consequently, given $x_1 \neq x_2$, and y_1, y_2 there is a polynomial, say $p(x)$ such that $p(x_j) = y_j$. Show this is true for distinct $x_1, ..., x_n$ and arbitrary $y_1, ..., y_n$.
3. Let K be compact in $\mathbb{R}$ and show $C(K)$ is separable. (Recall, $C(K)$ is a normed linear space with the sup norm).
4. Let $\mathcal{A}$ be an algebra of continuous functions on $\mathbb{R}$. Suppose we know two facts about $\mathcal{A}$:

   1. Given $x_1 \neq x_2$, and y_1, y_2 there is an $f \in \mathcal{A}$ such that $f(x_j) = y_j$.

   2. For every $f, g \in \mathcal{A}, \max\{f, g\} = f \vee g \in \mathcal{A}$ and $\min\{f, g\} = f \wedge g \in \mathcal{A}$.

   (a) Let K be compact in $\mathbb{R}$ and fix $y \in K$. Given a continuous function g and $\epsilon > 0$, show there is some $f \in \mathcal{A}$ such that $f(y) = g(y)$ and $f(x) > g(x) - \epsilon$ for every $x \in K$.

   (b) Show $\bar{\mathcal{A}} = C(K)$.