1. Let
$$f = \begin{cases} x^2 & \text{if } x \in \mathbb{R} - \mathbb{Q} \\ 0 & \text{else} \end{cases}$$
   (a) Is f continuous anywhere?
   (b) Is f differentiable anywhere?
   (c) Is f Riemann integrable on any interval?

2. Can you find a function from $[0,1] \to \mathbb{R}$ which has infinitely many discontinuities but is still Riemann integrable?

3. Notice for any function $f : [a,b] \to \mathbb{R}$ the upper integral, $\overline{\int} f$ is defined. Show that $|\overline{\int} f| \leq \overline{\int} |f|$. In particular, if $f \in \mathfrak{R}[a,b]$, $|\int f| \leq \int |f|$.

4. Show $f \in \mathfrak{R}[a,b] \iff f \in \mathfrak{R}[c,d]$ for every $[c,d] \subseteq [a,b]$.

5. Suppose $f : [0,1] \to \mathbb{R}$ is a bounded function such that for all a, b $\{x : a \leq f(x) < b\}$ is a union of disjoint intervals
   (a) Show that f is Riemann integrable.
   (b) Let $m\{x : k/N \leq f(x) < k/N\}$ be the sum of the lengths of the disjoint intervals of $\{x : k/N \leq f(x) < k/N\}$ (for example, $m([1/2, 3/4] \cup [7/8, 1)) = 1/4 + 1/8$.) Show
$$\sum_{k=0}^{N^2} \frac{k}{N} m\{x : k/N \leq f(x) < k/N\} \to \int f$$
   as $N \to \infty$.