ECE 302 Quiz 5
(10 points)

Name: Solution (3 points) 07/19/2016

1. Let $X$ and $Y$ be random variables with means $\mu_X$, $\mu_Y$, variances $\sigma_X^2$, $\sigma_Y^2$ and correlation coefficient $\rho_{XY}$. **Show all work. Express all answers in terms of the given parameters.**

   (a) (3 points) Find $\text{Cov}[X, Y]$.

   (b) (4 points) Find $\mathbb{E}[(aX + bY)^2]$, where $a$ and $b$ are constants.

(a) $\rho_{XY} = \dfrac{\text{Cov}[X,Y]}{\sqrt{\text{Var}[X]\,\text{Var}[Y]}}$

$\Rightarrow \text{Cov}[X,Y] = \boxed{\rho_{XY}\,\sigma_X\,\sigma_Y}$

(b) $E[(aX+bY)^2] = E[a^2X^2 + b^2Y^2 + 2abXY]$

$= a^2E[X^2] + b^2E[Y^2] + 2abE[XY]$

$= \boxed{\begin{array}{l} a^2(\sigma_X^2 + \mu_X^2) + b^2(\sigma_Y^2 + \mu_Y^2) \\ +2ab(\rho_{XY}\sigma_X\sigma_Y + \mu_X\mu_Y) \end{array}}$