8) $|E[X]| \le \sqrt{E[X^2]}$

Pf: Later...

E.g. A r.v. X has pdf

$$f_X(x) = e^{-(x-1)} \quad , \quad 1 < x < \infty$$
$$= 0 \quad , \quad \text{else}$$

a) Find the mean value of X

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$= \int_{1}^{\infty} x e^{-(x-1)}\, dx$$ integration by parts

$$= 2$$

b) Find Variance of X

$$Var[X] = E[X^2] - (E[X])^2$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int_{1}^{\infty} x^2 e^{-(x-1)}\, dx$$ integration by parts

$$= 5$$

$$Var[X] = 5 - 2^2 = 1$$

c) ~~$E[(X+3)^2]$~~ ,

$$Y = g(X) = (X+3)^2 = X^2 + 6X + 9$$

Find the mean of Y

$$E[Y] = E[(X+3)]^2$$

$$= E[(X^2 + 6X + 9)]$$

$$= E[X^2] + 6E[X] + 9$$

$$= 5 + 6 \cdot 2 + 9$$

$$= 26$$

Eg A r.v. X has pmf

$$p_X(x_i) = 0.1 \quad , \quad x_i = -2$$

$$= 0.2 \quad , \quad x_i = -1$$

$$= 0.3 \quad , \quad x_i = 0$$

$$= 0.4 \quad , \quad x_i = 1$$

a) Find the mean value of X

$$E[X] = \sum_{x_i} x_i \, p_X(x_i)$$

$$= (-2)(0.1) + (-1)(0.2) + 0(0.3) + 1 \cdot (0.4)$$

$$= 0$$

b) Find the variance of X

$$E[X^2] = \sum_{x_i} x_i^2 \, p_X(x_i)$$

$$= (-2)^2(0.1) + (-1)^2(0.2) + 0^2(0.3) + 1^2(0.4)$$

$$= 1$$

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$= 1 - 0^2 = 1$$

<u>Conditional cdf, pdf, and pmf</u>

We previously discussed how ~~the~~ knowledge of the occurrence of an event B can affect the likelihood of the occurrence of another event A.

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}, \quad \Pr(B) > 0$$

Can also consider how knowledge of an event M can affect the probability distribution of a r.v. X

Conditional pmf of a discrete random ~~var~~ variable X is

$$P_X(x_i \mid M) = Pr(X = x_i \mid M)$$

$$= \frac{Pr(\{X = x_i\} \cap M)}{Pr(M)} \quad , \; Pr(M) > 0$$

Conditional cdf of a r.v. X is

$$F_X(x \mid M) = Pr(X \leq x \mid M)$$

$$= \frac{Pr(\{X \leq x_i\} \cap M)}{Pr(M)} \quad , \; Pr(M) > 0$$

Conditional pdf of a r.v. X

$$f_X(x \mid M) = \frac{d}{dx} F_X(x \mid M)$$

$$= \frac{d}{dx} Pr(X \leq x \mid M)$$

In general will consider the case where M is determined by X. In other words

$M = \{X \in A\}$, A is a subset of $\mathbb{R}$

Then in this case

$$F_X(x \mid X \in A) = \Pr(X \le x \mid X \in A)$$

$$= \frac{\Pr(\{X \le x\} \cap \{X \in A\})}{\Pr(X \in A)}$$

$$= \frac{\int_{\{x' :\ x' \le x \text{ and } x' \in A\}} f_X(x')\,dx'}{\Pr(X \in A)}$$

$$= \frac{\int_{-\infty}^{x} f_X(x')\, \mathbb{1}_A(x')\,dx'}{\Pr(X \in A)}$$

$$\int_A f_X(x)\,dx \longrightarrow \Pr(X \in A)$$

where $\mathbb{1}_A(x) = 1$ if $x \in A$

$= 0$ else

$$f_X(x \mid X \in A) = \frac{d}{dx} F_X(x \mid X \in A)$$

$$= \frac{\frac{d}{dx} \int_{-\infty}^{x} f_X(x')\, \mathbb{1}_A(x')\,dx}{\Pr(X \in A)}$$

$$= \frac{f_X(x)\ \mathbb{1}_A(x)}{\Pr(X \in A)}$$

this is a truncated and scaled version of $f_X(x)$